Implementation and validation of a novel boundary condition sensitised to pressure gradient effects

Argyrios Apostolidis

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Abstract

Near-wall modelling has always been a major challenge in Computational Fluid Dynamics (CFD). Prediction of the characteristics of a wall-bounded flow imposes correct capturing and resolving of near-wall effects. The most accurate method available is integration of governing equations all the way to the wall. The turbulence model, in this case, has to include non-viscous wall-blocking and viscous damping effects, and the quality of the results heavily depends on the correct description of the model. The disadvantage of this method is the fine mesh resolution that is needed near wall, which leads to excessive computational costs.

A different approach may be the use of wall functions, which calculate the variables in the near-wall cell from simplified algebraic equations with respect to wall distance. When, such a method is employed, a coarse mesh is used and the first cell centre is in the fully turbulent region, not requiring near-wall effects to be resolved. However, the assumptions that lead to these simplified boundary conditions, proved to have limited use only in equilibrium flows.

Popovac [10] proposed a generalisation of the standard wall functions to include non-equilibrium effects by integrating analytically the parabolised moment equation and assuming a linear variation of turbulent viscosity.

Another problem in wall-treatment methods has been the a-priori knowledge of near-wall location inside the boundary layer, to provide the correct boundary condition. This is extremely difficult in flows which involve strong velocity gradients that can lead the same cell size lie either in viscous or the logarithmic sublayer. Popovac and Hanjalic [8] proposed a compound wall treatment method that combines wall integration with wall functions through a blending function proposed by Kader [6].

Purpose of this study has been to implement the above described wall treatments in OpenFOAM and validate their results against experimental data. A conclusion is reached that current wall functions are incapable to capture non-equilibrium effects and the generalised wall function combined with the compound wall treatment offer a significant advantage in such flows.
Acknowledgements

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I would like to express my deep gratitude to Prof. A. Kalfas for sharing with me his deep knowledge of Turbulence and always reminding me that computational fluid dynamics is an approximation that can not be treated as a ‘gospel’. Pushing me this-way to contribute to the development of CFD towards results that can better represent reality.

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Argyris Apostolidis
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'Big whorls have little whorls, which feed on their velocity.
And little whorls have lesser whorls. And so on to viscosity.'

*Lewis Fry Richardson*
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Chapter 1

Introduction

1.1 Motivation and Objectives

Understanding and predicting turbulent flow phenomena is vital in a variety of engineering problems from blood flow prediction to airplane design. However, the non-deterministic nature of turbulence makes it extremely difficult to accurately simulate it. In addition, the largest part of the flow is heavily affected by the near-wall region, called the sublayer. The most accurate way to deal with wall bounded flows is to integrate the RANS equations all the way to the wall, including the effects of non-viscous wall-blocking and viscous damping. At the same time an adequate dense grid, must be provided, in order to capture the high gradients that are dominating this thin region. Such an approach leads to increased computational power that can be proved as a limitation in highly complex cases, especially in industrial applications, where the key priority is the decrease of development time and designing period of a new product.

A different approach that can economically, represent the near-wall region was firstly presented by Patankar and Spalding [11]. The standard wall functions employ simple algebraic relations to determine the flow quantities at the first node, that must lie on the logarithmic region ($y^+ > 30$). Chieng and Launder [1] wanted to improve the standard wall functions by introducing a two-layer approach splitting the first cell into the viscous sublayer and the inertial sublayer, averaging the quantities of interest over these two distinct regions. Craft further developed
this idea by assuming the eddy viscosity variation across the first cell and by including non-equilibrium effects such as convection and pressure gradient for his Wall Function. Finally, Popovac and Hanjalic [8] simplified the wall functions developed by Craft, so they can be, easily, implemented into any code while they also provided a compound wall treatment method that provides adequate results in the whole boundary layer region.

1.2 Outline of the thesis

The thesis is outlined as follows. In Chapter 2 the background theory of turbulence and turbulence modelling is outlined, including the equations governing the fluid motion as well as the reason for near wall modelling. Chapter 3 presents various wall boundary condition approaches, their assumptions, advantages and disadvantages. Chapter 4 focuses in the simulation software used in this study, OpenFOAM. In Chapter 5 the three validation studies for the selected wall wall treatment method are outlined, along with its results and comparison between different approaches. Finally, Chapter 7 summarises the concluding remarks and provides recommendations for future studies.


Chapter 2

Theoretical Background

2.1 Introduction

Turbulence is the chaotic fluid motion, characterized by disorder and diffusivity. The irregular variations of velocity and pressure about some mean value, lead to intensive unsteadiness even with constant imposed boundary conditions. The Reynolds number of a flow field is the ratio between the inertia forces and viscous forces. If this ratio is below some critical value $Re_{crit}$ the viscous effects damp instabilities so the flow field becomes steady, and the laminar flow can maintain itself. Above the $Re_{crit}$ the inertial forces provide enough kinetic energy to change radical the flow field, which becomes turbulent. In Fig. 2.1 the velocity fluctuations underline the random nature of turbulence.

![Figure 2.1: Velocity fluctuations in a turbulent flow field](image)

Figure 2.1: Velocity fluctuations in a turbulent flow field
Such a flow can be prescribed using the Reynolds decomposition method, which decomposes an arbitrary variable $\phi$ into its steady mean value $\Phi$ and its fluctuation $\phi'$.

$$\phi(t) = \Phi + \phi'(t)$$  \hspace{1cm} (2.1)

## 2.2 Governing flow equations

The motion of an incompressible Newtonian fluid is described by the Navier-Stokes Equation, which consist of the continuity equation in Cartesian coordinates using the Einstein notation:

$$\frac{\partial u_i}{\partial x_i} = 0$$  \hspace{1cm} (2.2)

and the momentum equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j}$$  \hspace{1cm} (2.3)

where $u_i$ and $p$ are the instantaneous velocity and pressure respectively, $\rho$ is the fluid density and $\nu$ the molecular viscosity. The method, where the above described equations are directly solved without using any turbulence model, is called Direct Numerical Simulation (DNS) and it is the most accurate simulation, although computational too expensive even for the fastest supercomputers.

Therefore a simpler approach utilizes the Reynolds decomposition method and leads to the Reynolds averaged Navier-Stokes equations (RANS):

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_i \partial x_j} - \frac{\partial u_i u_j}{\partial x_j}$$  \hspace{1cm} (2.4)

The additional terms, $-u_i u_j$, that have been created during the averaging process are called Reynolds Stresses and include the effect of turbulence in the RANS equations.
2.3 Closure Problem

This way an extra set of unknown variables are formed with no further equations, which is known as the closure problem. In order to overcome this and close the set of equations by Eq. (2.4) etc., the Reynolds Stresses have to be approximated with known flow quantities. The most common way follows the hypothesis of Boussinesq, by introducing the concept of eddy viscosity $\nu_t$.

$$-\overline{u_i u_j} = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \quad \text{(2.5)}$$

where $k = \frac{1}{2} \overline{u_i u_i}$ is the kinetic energy of turbulence. The base of the hypothesis is that the chaotic nature of a turbulent flow intensifies the transportive and diffusive phenomena, something that can be also achieved by increasing the diffusion coefficient. The eddy viscosity, added to the molecular viscosity, represents this extra contribution of the turbulent flow.

The Boussinesq hypothesis assumes an isotropic nature of turbulence, which is not the case in various flows. In a simple shear flow, for example, the Reynolds stresses close to the wall using this hypothesis are $\overline{u_1 u_1} = \overline{u_2 u_2} = \overline{u_3 u_3} = 2k/3$ whereas experiments and DNS studies suggest an anisotropy in the normal stress distribution with $0.5 \overline{u_1 u_1} \approx \overline{u_2 u_2} \approx \overline{u_3 u_3}$. In most of the cases this may not impose a problem in predicting the overall flow field, however in cases of secondary, swirling and separated flows can lead to wrong predictions.

2.4 Mixing length model - Law of the Wall

Using Boussinesq’s hypothesis, Ludwig Prandtl proposed that the Reynolds stresses are produced by normal momentum transfer from high to low momentum regions. The mixing length, $l_m$, is the distance over which a fluid element transfers momentum, analogous to the mean free path in kinetic theory, resulting in the following expression for the Reynolds stresses:
\[ \bar{u}_1u_2 \approx l_m v_m \frac{\partial U}{\partial y}, \quad l_m = \kappa y \quad (2.6) \]

where \( v_m \) is the mixing velocity, which Prandtl proposed to be:
\[ v_m = a_l m \frac{\partial U}{\partial y}. \]

Including the expression for the mixing velocity Eq. (2.6) transforms into:
\[ \bar{u}_1u_2 = \kappa^2 y^2 \left( \frac{\partial U}{\partial y} \right)^2 \quad (2.7) \]

We consider the steady-state RANS equations for a fully-developed Couette flow in which the streamwise gradients of velocity and the wall normal velocity are zero leading to the simplified \( U \)-velocity equation:
\[ \frac{\partial}{\partial y} \left( \mu \frac{\partial U}{\partial y} - \rho \bar{u}_1u_2 \right) = 0 \quad (2.8) \]

Integrating and using as boundary conditions that the Reynolds stress at the wall is zero and \( \tau = \mu \frac{\partial U}{\partial y} = \tau_w \), one obtains the shear stress at the wall:
\[ \mu \frac{\partial U}{\partial y} - \rho \bar{u}_1u_2 = \tau_w \quad (2.9) \]

From Eq. (2.8) and Eq. (2.9) it is apparent that the total shear shear stress is constant over the extent of the wall layer and equal to the wall value \( \tau_w \).

Inserting in Eq. (2.9) the mixing length approximation leads to:
\[ \mu \frac{\partial U}{\partial y} - \kappa^2 y^2 \left( \frac{\partial U}{\partial y} \right)^2 = \tau_w \quad (2.10) \]

At small wall distances, \( 0 < y^+ < 5 \) the velocity fluctuations go to zero due to damping so the viscous effects dominates the flow, and the solution for the viscous sublayer is formed:
\[ \mu \frac{\partial U}{\partial y} = \tau_w \Rightarrow U = \frac{\tau_w}{\rho \nu} y \quad (2.11) \]
2.5 The $k - \varepsilon$ model

Significantly out from the wall, at wall distances $30 < y^+ < 100$ the Reynolds stresses have far greater impact than the viscous stresses, that can be neglected:

$$\kappa^2 y^2 \left( \frac{\partial U}{\partial y} \right)^2 = \frac{\tau_w}{\rho} \Rightarrow \frac{\partial U}{\partial y} = \frac{1}{\kappa} \sqrt{\frac{\tau_w}{\rho}} y^{1/2}$$

(2.12)

Integrating Eq. (2.12) gives the logarithmic velocity profile:

$$U = \frac{1}{\kappa} \sqrt{\frac{\tau_w}{\rho}} \ln y + c$$

(2.13)

Dividing Eq. (2.11) and Eq. (2.13) with the friction velocity, $u_\tau = \sqrt{\tau_w/\rho}$, leads to the dimensionless form, known as the law of the wall:

$$U_\text{vis}^+ = y^+,$$

(2.14a)

$$U_\text{log}^+ = \frac{1}{\kappa} \ln Ey^+$$

(2.14b)

The two regions are shown in Fig. 2.2 joined with the buffer layer ($5 < y^+ < 30$), for which no simple analytic expressions exists, and where the viscous and turbulent effects have the same contribution. For wall non-dimensionalized distances above $y^+ = 100$ the velocity-defect law is applied, that strongly depends on the flow characteristics, so it can not be described by a "universal" law.

2.5 The $k - \varepsilon$ model

The $k - \varepsilon$ is the most common RANS turbulence model used in CFD. In OpenFOAM a variant of the standard $k - \varepsilon$ proposed by Launder and Spalidng that incorporates Rapid Distortion Theory (RDT) contribution as described by El Tahry, is used. Eddy viscosity is obtained using:

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}$$

(2.15)
Figure 2.2: Algebraic relation for viscous and logarithmic layers and a DNS from

The $k$ transport equation can be derived from the Reynolds stress equation and is given by:

$$\frac{D}{Dt}(\rho k) = \nabla \cdot (\rho D_k k) + G_k - \frac{2}{3} \rho (\nabla u) k - \rho \varepsilon + S_k$$  (2.16)

and the dissipation rate by:

$$\frac{D}{Dt}(\rho \varepsilon) = \nabla \cdot (\rho D_\varepsilon \nabla \varepsilon) + \frac{C_1 G_k \varepsilon}{k} - \left( \frac{2}{3} C_1 + C_{3,RDT} \right) \rho (\nabla u) \varepsilon - C_2 \rho \frac{\varepsilon^2}{k} + S_\varepsilon$$  (2.17)

where the default model coefficients are given in the following table:

<table>
<thead>
<tr>
<th>$C_\mu$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_{3,RDT}$</th>
<th>$\sigma_k$</th>
<th>$\sigma_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>1.44</td>
<td>1.92</td>
<td>-0.33</td>
<td>1</td>
<td>1.3</td>
</tr>
</tbody>
</table>

The boundary conditions at the wall are specified by different wall functions as will be described in the upcoming paragraphs.
2.6 Near Wall Reynolds Stress behavior

Adjacent to the wall, the variation of velocities fluctuations can be obtained from the Taylor series expansions as follows:

\[ u_i(y) = a_i + b_i y + c_i y^2 + ... \]  \hspace{1cm} (2.18)

where \( y \) the wall normal distance and coefficients \( a_i, b_i \) and \( c_i \) random functions of \( x, z \) and time. The no slip condition at the wall imposes that the mean velocity as well as the velocity fluctuations are zero, it follows that \( a_1 = a_2 = a_3 = 0 \). Since the continuity equation is also valid for the velocity fluctuations of an incompressible fluid:

\[ \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} = 0 \] \hspace{1cm} (2.19)

However, since \( u_1 \) and \( u_3 \) are zero for all \( x \) and \( z \), the wall gradients will be also zero \( \left( \frac{\partial u_1}{\partial x} \right)_{y=0} = 0 \) and \( \left( \frac{\partial u_3}{\partial z} \right)_{y=0} = 0 \). Hence, from Eq. (2.19) the wall normal gradient must be zero \( \left( \frac{\partial u_2}{\partial y} \right)_{y=0} = 0 \). This results that the \( b_2 \) coefficient is zero. From the above expressions it is evidently that while the \( u_1 \) and \( u_3 \) vary linearly, with respect to normal wall distance, the normal component \( u_2 \) varies with the quadratic of the distance.

From Eq. (2.18) the assymptotes of the turbulent stresses can be obtained:

\[ \overline{u_1u_1} = b_1^2 y^2 + c_1 y^4 + ... \]
\[ \overline{u_2u_2} = c_2 y^4 + ... \]
\[ \overline{u_3u_3} = b_3^2 y^2 + c_3 y^4 + ... \]
\[ \overline{u_1u_2} = d_4 y^3 + ... \] \hspace{1cm} (2.20)

i.e \( \overline{u_1u_2} \) vary with the cube of the wall distance, which means that for small distances the turbulent shear stresses will have a negligible effect compared to viscous stresses, whereas at larger distances the turbulent effects will dominate the flow, as also seen in Fig. 2.3.

The turbulent kinetic energy, \( k \), close to the wall is given by:
Figure 2.3: Turbulent, viscous and total shear stress across fully developed pipe flow.

\[
\begin{align*}
    k &= \frac{1}{2} (u_1^2 u_1 + u_2^2 u_2 + u_3^2 u_3) \\
    &= \frac{1}{2} (Ay^2 + By^3 + Cy^4) \\
    &\approx \nu (A^2 + C^2) \\
\end{align*}
\]

and the dissipation rate, \( \varepsilon \):

\[
\varepsilon = \nu \left( \frac{\partial u_i}{\partial x_j} \right)^2
\]

\[
\approx \nu (A^2 + C^2)
\]

where \( A \) and \( C \), constants calculated from the coefficients \( a_i, b_i \) and \( c_i \).

At the wall \( y = 0 \) after combining Eq. (2.21) and Eq. (2.22) the following expression for the dissipation rate is obtained:

\[
\varepsilon_w = \frac{2\nu k}{y^2}
\]

which is often used as boundary condition for \( \varepsilon \).

A similar distribution across a zero pressure gradient boundary layer can be expressed for the turbulent kinetic energy production \( P_k \). Since the total shear stress in the logarithmic area of the boundary layer stays constant and the wall normal mean-velocity gradient decreases as
\( \frac{\partial U}{\partial y} \propto y^{-1} \), the production rate \( (P_k = u_1 u_2 \partial U/\partial y) \) must also decrease. However, at the wall \( u_1 u_2 = 0 \), therefore there must be a point in the buffer region where the production of turbulent kinetic energy reaches its maximum value. This inflection point occurs when:

\[
\frac{\partial}{\partial y} \left( \frac{u_1 u_2}{u_1 u_2} \frac{\partial U}{\partial y} \right) = 0 \tag{2.24}
\]

expansion of the differential form leads to:

\[
\frac{u_1 u_2}{u_1 u_2} \frac{\partial^2 U}{\partial y^2} + \frac{\partial U}{\partial y} \frac{\partial u_1 u_2}{\partial y} = 0 \tag{2.25}
\]

As mentioned previously, the total shear stress remains approximately constant, therefore we can replace the derivative of turbulent shear stress with the derivative of the viscous stress. Using this assumption, the inflection point emerges to be the point where the viscous stresses equal the turbulent ones \( (-u_1 u_2 = \nu \partial U/\partial y) \).

The DNS data form Hoyas and Jiminez(2006), as shown in Fig. 2.4, for a plane channel flow, suggest that for \( (y+ > 30) \) there is a local equilibrium of turbulent kinetic energy production and dissipation \( (P_k = \varepsilon) \). In the buffer region \( (5 < y+ < 30) \) viscous and turbulent diffusion terms have a significant effect and finally in the viscous sub-layer \( (y+ < 5) \) dissipation and viscous diffusion cancel each other.
Figure 2.4: Turbulent kinetic energy equation budget in a plane channel flow taken from DNS of Hoyas and Jimenez (2006). $P_k$ = production; $\varepsilon$ = dissipation rate; $D_P^k$ = velocity-pressure gradient term; $D^l_k$ = turbulent transport; $D^\nu_k$ = viscous diffusion.
Chapter 3

Wall Functions

Most turbulent flows exhibit a similar behavior near a solid boundary. The no slip boundary condition at the wall contributes to the development of a boundary layer. In this region there is a strong variation of the mean velocity normal to the wall, requiring very fine grids to provide adequate numerical resolution. There are two main strategies for modelling the near-wall phenomena:

- **Wall Integration**, where the whole boundary layer is resolved numerically, while modifying the turbulence model to account for viscous and non viscous damping of turbulence near the wall.

- **Wall Functions**, where the boundary layer is modeled through simplified algebraic relations.

Turbulence models that are using the first method are called Low-Reynolds number (LRN) turbulence models and are computational expensive, but accurate. While the wall function approach is used by the High-Reynolds number (HRN) turbulence models providing faster but less accurate solutions.
### 3.1 Standard Wall Functions

The most common wall functions use the findings of the law of the wall, as described in Section 2.4, are called Standard Wall Functions (SWF). The velocity, however, is non-dimensionlized using a new velocity scale: \( u_k = C_{\mu}^{1/4} k^{1/2} \), instead of the friction velocity \( u_\tau \). The reasoning behind this change, has to do with the improvement in performance in separation and reattachment points, where the wall shear stress tends to zero, and is in agreement with Townsend’s observation: \( \tau_w/\rho = C_{\mu}^{1/2} k \).

From Eq. (2.13) and using the new velocity scale, the wall shear stress is computed from:

\[
\frac{\tau_w}{\rho} = \frac{\kappa u_k U_p k_p^{1/2}}{\ln (E y_p^*)},
\]

(3.1)

where \( y_p^* = C_{\mu}^{1/4} k_p^{1/2} y/\nu \) and \( k_p \) is computed at the near wall cell center.

Reformulating the standard wall shear stress expression:

\[
\frac{\tau_w}{\rho} = \nu_{eff} u_p y_p
\]

(3.2)

where \( \nu_{eff} = \nu + \nu_t \) is the effective turbulent viscosity.

Combining Eq. (3.1) and Eq. (3.2), the boundary condition for the turbulent viscosity (\( \nu_t \)) is formed:

\[
\nu_t = \nu \left( \frac{\kappa y_p^*}{\ln (E y_p^*)} - 1 \right),
\]

(3.3)

Turbulent kinetic energy at the wall node is calculated from the transport equation, after deleting the diffusion term and modifying the production term to be in equilibrium with the dissipation rate at the first cell.
$$P = \varepsilon = \nu_t \left( \frac{\partial U}{\partial y} \right)^2 = \frac{C_{1/4}^{3/4} C_{1/2}^{3/2}}{\kappa y_p}$$ (3.4)

In OpenFOAM, the production of turbulent kinetic energy is given by:

$$P = (\nu + \nu_t) \left| \frac{dU}{dy} \right| \frac{C_{1/4}^{1/2} C_{1/2}^{1/2}}{\kappa y_p}$$ (3.5)

## 3.2 Wall Functions without the law of the wall

The above described standard wall functions are a result of several assumptions that have been made in order to find simple algebraic formulas describing for the flow near the wall. However such assumptions limit their applications into simple flows that do not exhibit variations in pressure gradient or other more complex phenomena. For this reason there has been an effort from various authors to generalize the concept of wall functions.

### 3.2.1 Non Equilibrium Wall Function

The Non-Equilibrium Wall Functions that have been developed in 1995 by S. E. Kim and D. Choudhury are an extension of the law of the wall sensitized to pressure gradient effects, and have been implemented in the commercial code of Fluent.

According to Fluent User’s Guide the wall tangential velocity modified for accounting the pressure gradient is:

$$\frac{\tilde{U} C_{1/4}^{1/2} k_{1/2}}{\tau_w / \rho} = \frac{1}{\kappa} \ln \frac{\rho C_{1/4}^{1/2} k_{1/2} y_p}{\mu}$$ (3.6)

where

$$\tilde{U} = U - \frac{1}{2} \frac{dp}{dx} \left[ \frac{y_{\nu}}{\rho \kappa \sqrt{k}} \ln \frac{y_{\nu}}{y} + \frac{y - y_{\nu}}{\rho \kappa \sqrt{k}} + \frac{y_{\nu}^2}{\mu} \right]$$ (3.7)
and \( y_\nu \) is the physical viscous sublayer thickness, computed from:

\[
y_\nu = \frac{\mu y_\nu^*}{\rho C_\mu^{1/4} k_p^{1/2}} \tag{3.8}
\]

where \( y_\nu^* = 11.225 \). For taking account the variation of turbulent energy production and dissipation near the wall, the non-equilibrium wall functions use the two layer concept. The main idea is that the wall adjacent cells consist of a viscous sublayer and a fully turbulent layer, and the variations of wall shear stress, turbulent kinetic energy and turbulent dissipation are the following:

\[
\tau_w = \begin{cases} 
0, & y < y_\nu \\
\tau_w, & y > y_\nu 
\end{cases} \tag{3.9}
\]

\[
k = \begin{cases} 
\left( \frac{y}{y_\nu} \right)^2 k_p, & y < y_\nu \\
k_p, & y > y_\nu 
\end{cases} \tag{3.10}
\]

\[
\varepsilon = \begin{cases} 
\frac{2\nu k}{y^2}, & y < y_\nu \\
\frac{k^{3/2}}{C_l^* y}, & y > y_\nu 
\end{cases} \tag{3.11}
\]

where \( C_l^* = \kappa C_\mu^{-3/4} \).

Using the above profiles the cell average production and dissipation can be computed:

\[
\overline{P_{k,P}} = \frac{1}{y_n} \int_0^{y_n} \tau_w \frac{\partial U}{\partial y} dy = \frac{\tau_w^2}{\rho \kappa y_n C_\mu^{1/4} k_p^{2/3}} \ln \frac{y_n}{y_\nu} \tag{3.12}
\]

\[
\varepsilon = \frac{1}{y_n} \int_0^{y_n} \varepsilon dy = \frac{1}{y_n} \left[ \frac{2\nu}{y_\nu} + \frac{k_p^{1/2}}{C_l^*} \ln \frac{y_n}{y_\nu} \right] \tag{3.13}
\]

where \( y_n = 2y_p \).
3.2.2 Analytical Wall Function

In the scheme proposed by Gerasimov and Craft the turbulent viscosity is zero inside the viscous sublayer, while outside of it increases linearly with the distance from the wall, as shown in the below:

\[
\frac{\mu_t}{\mu} = \begin{cases} 
0, & y < y_v \\
\kappa^*(y^* - y_v^*), & y > y_v
\end{cases}
\]  

(3.14)

where \( \kappa^* = c_c \mu_c \) and \( y^* = \rho k^{1/2} y / \mu \).

Purpose of this wall function is to derive a simple form of the momentum equation in order to integrate it analytically. Ignoring the diffusion parallel to the wall and convection normal to the wall the x-momentum equation becomes:

\[
\frac{\partial}{\partial y^*} \left[ (\mu + \mu_t) \frac{\partial U}{\partial y^*} \right] = \frac{\nu^2}{k_p} \left[ \rho U \frac{\partial U}{\partial x} + \frac{\partial P}{\partial x} \right] c_U
\]  

(3.15)

Eq. (3.15) can then be integrated first inside the viscous sublayer and then across the turbulent layer, by assuming continuity across the interface of the velocity and its normal gradient, leads to the velocity profile expression:

\[
U = \begin{cases} 
\frac{C_U}{2 \mu} y^{*2} + \frac{A_U}{\mu} y^* + B_U, & y < y_v \\
\frac{C'_U}{\alpha \mu} y^* + \left[ \frac{A'_U}{\alpha} - \frac{C'_U}{\alpha^2 \mu} (1 - \alpha y_v^*) \right] \ln [1 + \alpha (y^* - y_v^*)] + B'_U & y > y_v
\end{cases}
\]  

(3.16)

Constants \( A_U, B_U, A'_U \) and \( B'_U \) can be determined by applying boundary conditions at the wall and at the edge of the first cell \( (y_v) \).

Then the wall shear stress can be computed from:

\[
\tau_w = \mu \left. \frac{\partial U}{\partial y} \right|_w = \mu k_p^{1/2} \left. \frac{\partial U}{\partial y} \right|_w = \frac{k_p^{1/2} A_U}{\nu}
\]  

(3.17)
The local generation of turbulent kinetic energy \( P_{P,k} = \nu_t (dU/dy)^2 \) can be integrated over the near wall cell and give us the average value for solving the k equation.

For the dissipation rate \( \varepsilon \) the following variation is assumed:

\[
\varepsilon = \begin{cases} 
2\nu k_p/y_d^2, & y < y_d \\
 k_p^{1.5}/(c_l y), & y > y_d 
\end{cases}
\] (3.18)

where \( y_d = 2c_l \) is the characteristic dissipation scale, which ensures continuity. Then the dissipation rate can be integrated over the cell to produce the cell averaged value.

The AWF has behaved reasonably well in cases of a flow down a pipe, a mixed convection flow and an impinging jet flow. However, the model did not gain widespread use due to its complex implementation in unstructured grids and sometimes bad stability. Furthermore, it can not be generalized easily to other turbulence models and some of its features are \textit{ad hoc}.

### 3.2.3 Generalised wall function

Popovac and Hanjalic have proposed a simpler version of AWF similar to the standard wall functions. A different effective viscosity profile variation has been applied that allows a straight-forward integration. In the viscous region of the boundary layer the effective viscosity equals that of molecular viscosity. For the turbulent region the molecular viscosity can be ignored and a linear variation of the turbulent viscosity is assumed, following observations by DNS data from Popovac:

\[
\frac{\mu_{eff}}{\mu} = \begin{cases} 
1, & y < y_\nu \\
\kappa^* y^*, & y > y_\nu 
\end{cases}
\] (3.19)

where \( \mu_{eff} = \mu_t + \mu \).
The wall parallel momentum equation can be written as:

\[
\frac{\partial}{\partial y} \left[ (\mu + \mu_t) \frac{\partial U}{\partial y} \right] = \left[ \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} + \frac{\partial P}{\partial x} \right]
\]

(3.20)

The term \( C_U \) are considered to be constant across the near-wall cell, and known from the previous iteration.

In order to obtain the gradient of velocity, integration of Eq. (3.20) leads to:

\[
\mu_{eff} \frac{dU}{dy} = C_U y + A
\]

(3.21)

where A is the integration constant equals with the wall shear stress \( \tau_w \).

Integration across the viscous sublayer yields:

\[
U_1 = \frac{C_U y^2}{2\mu} + \frac{A_1 y}{\mu} + B_1
\]

(3.22)

whereas across the turbulent regions gives:

\[
U_2 = \frac{C_U y}{\rho \kappa u_r} + \frac{A_2}{\rho \kappa u_r} \ln(y) + B_2
\]

(3.23)

The integrating constants can be computed after ensuring continuity and smoothness of the velocity distribution and after some rearrangement the expression for the wall shear stress can
Chapter 3. Wall Functions

be obtained:

\[ \tau_w = \frac{\rho U}{\mu} \left( \frac{C_U y}{\kappa u_*} \right) - \frac{\rho y}{\mu} + \ln \left( \frac{y}{\kappa u_*} \right) \]  

(3.24)

the above equation yields a similar to logarithmic velocity-law algebraic form for the velocity profile:

\[ U_p^* = \frac{1}{\kappa^* \psi} \ln \left( E^* y_p^* \right) \]  

(3.25)

where

\[ \psi = 1 - \frac{C_U y_p}{\rho \kappa c_p^{1/4} \kappa^{1/2} U_p} \]  

(3.26)

is the non-equilibrium correction factor and \( E^* \approx 4.8 \). For \( \psi = 1 \) the equations reduce to standard wall functions.

The generalized wall function has very similar approach to the standard wall functions and can be easily implemented in any commercial CFD code. It produces significantly more accurate results in cases such as backward facing step, impinging jet and flat plate boundary layer, than the standard law of the wall. However, the effective viscosity variation that is assumed for convenience in the integration, is not continuous which is physically unrealistic.

3.2.4 Compound Wall Treatment

The wall treatments that have been currently discussed dictate that the position of the near-wall cell must be known in advance. Either it will lie on the viscous sublayer, so a wall integration model is needed, or in the turbulent region, where the previously wall functions can be applied. This is rather difficult, especially in complex geometries or when there are strong variations in the velocity magnitude due to acceleration, deceleration or separation, which can lead to two same size cells requiring different wall boundary condition approaches.

Popovac and Hanjalic (2006a) and Popovac and Hanjalic (2005b) proposed an elliptic blending function that can overcome this problem, called Compound Wall Treatment (CWT). The method involves of combining the limiting values of the viscous and turbulent regions to pro-
duce a continuous function that can give results across the whole boundary layer (viscous, buffer and turbulent regions).

Various methods exist that provide similar blending across the boundary layer. The easiest being a simple switching formula taking the maximum value of the two regions:

\[ \phi_P = \max(\phi_P^\nu, \phi_P^t) \] (3.27)

where \( \phi \) is the variable in question, the superscript "\( \nu \)" denotes the viscous value "\( t \)" the fully turbulent value. Esch and Menter (2003) proposed a better blending function, using the quadratic mean:

\[ \phi_P = \sqrt{(\phi_P^\nu)^2 + (\phi_P^t)^2} \] (3.28)

However, both of the above expressions lack a physical rationale leading to erroneous behaviour. Popovac and Hanjlic used the idea proposed by Kader (1981), and formulated the exponential blending function as:

\[ \phi_P = \phi_P^\nu e^{-\Gamma} + \phi_P^t e^{-1/\Gamma} \] (3.29)

where \( \Gamma \) is a function of the normalised wall distance:

\[ \Gamma = \frac{0.01y^+}{1 + 5y^+} \] (3.30)

Application of the CWT for the wall shear stress and using the expression from the Generalised Wall Function in Eq. (3.24) yields:

\[ \tau_w = \tau_w^\nu e^{-\Gamma} + \tau_w^t e^{-1/\Gamma} = (\mu e^{-\Gamma} + \mu_w^{eff} \psi e^{-1/\Gamma}) \frac{U_P}{y_P} \] (3.31)
A-priori analysis in a plane channel flow in Fig. 3.2 shows the better results of the CWT compared with the other blending methods, apart from a small region around $y^+ \approx 7$.

Figure 3.2: A-priori analysis of different blending methods for the wall shear stress, for a plane channel flow at $Re_\tau = 800$.

The velocity profile across the boundary layer is obtained through the combination of Eq. (3.25) and CWT:

$$U^* = y^* e^{-1}\Gamma + \frac{1}{\kappa^* \psi^*} \ln E^* y^* e^{-1/\Gamma}$$  \hspace{2cm} (3.32)

Comparison between DNS data of Spalart (1988) in Fig. 3.3 for a flat plate, suggests the CWT provides an excellent velocity profile and can be used for velocity predictions irrespective of the wall-distance of the first node.

The same procedure is followed for obtaining the turbulent kinetic energy production $\mathcal{P}$ and the dissipation rate $\varepsilon$ from expressions discussed in previous chapters:

$$\mathcal{P} = C_\mu \frac{k^2}{\varepsilon} \left( \frac{dU}{dy} \right)^2 e^{-1}\Gamma + (\nu + \nu_t) \left| \frac{dU}{dy} \right| \frac{C_\mu^{1/4} k_{p}^{1/2}}{\kappa y_p} e^{-1/\Gamma}$$  \hspace{2cm} (3.33)

$$\varepsilon = \frac{2\nu k_P}{y_p^2} e^{-1}\Gamma + \frac{C_\mu^{3/4} k_P^{3/2}}{\psi \kappa y_p} e^{-1/\Gamma}$$  \hspace{2cm} (3.34)
where $\Gamma_\varepsilon = 0.001 y^+/(1 + y^+)$ is a modified blending parameter due to the strong variation of $\varepsilon$ near the wall.
Chapter 4

OpenFOAM

4.1 An Introduction to *OpenFOAM*

OpenFOAM (Open Source Field Operation and Manipulation) is a free and open source CFD software written in C++ and produced by *OpenCFD Ltd.*. It features a range of numerical schemes, methods and turbulence models that, due to its open source nature, can be completely customized and extended. Another advantage of OpenFOAM is its syntax for tensors operations and partial differential equations that closely resembles the equations being solved. For example the incompressible Navier-Stokes equations:

\[
\frac{\partial \rho U}{\partial t} + \nabla \cdot \phi U - \nabla \cdot \mu \nabla U = -\nabla p \tag{4.1}
\]

are represented by the code:

```cpp
solve
(
    fvm::ddt(rho, U)
    + fvm::div(phi, U)
    - fvm::laplacian(mu, U)
) - gradient(p)
```

(4.1)
OpenFOAM employs the finite volume method (FVM) for providing a numerical description of the domain, including the positions of points in which the solution is obtained and the description of the boundary. The FVM subdivides the flow domain into a finite number of smaller control volumes. Furthermore, if unsteady phenomena are taking place, the time domain is divided in small time steps.

4.2.1 Space Discretisation

The discretised cells of the solution domain are contiguous, meaning they do not overlap and completely fill the domain. The variables of interest are stored in the cell-center or on faces or vertices. A cell is bounded by an unrestricted number of faces, which can be called “arbitrarily unstructured”. Two neighbouring cells share a common face, called an “internal face”, while a face belonging only to one cell is called a “boundary face”. In Fig. 4.1 the common features of the FVM are shown, where $P$ and $N$ are the cell centres, $f$ is the internal face, $S_f$ the normal vector, which also represents the area of the face and $d$ the vector joining the cell centres.
4.2.2 Equation Discretisation

The goal is to express the partial differential equations into algebraic expressions in the following form:

\[
Ax = b
\]  

(4.2)

where \( A \) is a square matrix constituted from the implicit terms of the discretisation process, \( x \) the column vector of dependent variable and \( b \) is the source vector constituted from the explicit terms.

FVM discretisation of each term starts with first integrating over the cell volume \( V \) and using Gauss’s theorem converting volume integrals to surface integrals over the cell surface \( S \):

\[
\int_V \nabla \phi = \int_S dS \phi
\]  

(4.3)

The Diffusion Term

The diffusion term of a generic variable \( \phi \) integrated over a control volume and linearised as follows:
\[ \int_V \nabla \cdot (\Gamma \nabla \phi) dV = \int_S dS \cdot (\Gamma \nabla \phi) = \sum_f \Gamma_f S_f \cdot (\nabla \phi)_f \quad (4.4) \]

The Convection Term

The convection term is integrated over a control volume and linearised as follows:

\[ \int_V \nabla \cdot (\rho U \phi) dV = \int_S dS \cdot (\rho U \phi) = \sum_f S_f \cdot (\rho U_f) \phi_f \quad (4.5) \]

The face field \( \phi_f \) can be evaluated using a variety of schemes: **Central differencing (CD)**, **Upwind differencing (UD)** and the combination of CD and UD the **Blended differencing (BD)**.

The Gradient Term

The gradient term is computed using the Gauss theorem to the volume integral:

\[ \int_V \nabla \phi dV = \int_S dS \phi = \sum_f S_f \phi_f \quad (4.6) \]

4.2.3 Temporal Discretisation

The descretisation of time domain has also a variety of methods:

- **Euler implicit** is first order accurate in time, guarantees boundedness and is unconditionally stable, while taking only current values \( \phi^n \).

- **Explicit** is also first order accurate and guarantees boundedness, however is unstable if the Courant number, \( Co = U_f d / |d|^2 \Delta t \), is greater than one. It only takes values from the previous timestep \( \phi^0 \)

- **Crank Nicholson** is second order accurate, unconditionally stable, but does not guarantee boundedness. It takes the mean of the current and old values.
4.3 Solution Procedure

The numerical procedure to solve the Navier-Stokes equations used in this study is the SIMPLE algorithm for Semi-Implicit Method for Pressure Linked Equations. We recall the Navier-Stokes equations for an incompressible fluid are:

\[ \nabla \cdot \mathbf{U} = 0 \quad (4.7a) \]
\[ \nabla \cdot (\mathbf{U} \mathbf{U}) - \nabla \cdot (\nu \nabla \mathbf{U}) = -\frac{1}{\rho} \nabla p \quad (4.7b) \]

There are two main obstacles in solving these equations, firstly the non-linear term \( \nabla \cdot (\mathbf{U} \mathbf{U}) \) and secondly due to the non-existence of an explicit equation for the pressure.

The discretised momentum equation can be expressed in the form:

\[ A \cdot \mathbf{U} = \mathbf{H(U)} - \nabla p \quad (4.8) \]

solving for the velocity yields:

\[ \mathbf{U} = \frac{\mathbf{H(U)}}{A} - \frac{1}{A} \nabla p \quad (4.9) \]

and finally interpolation on cell faces gives:

\[ \mathbf{U}_f = \left( \frac{\mathbf{H(U)}}{A} \right)_f - \left( \frac{1}{A} \nabla p \right)_f \quad (4.10) \]

where \( A \) contains the diagonal matrix coefficients and \( \mathbf{H(U)} \) contains all off-diagonal matrix coefficients multiplied by their corresponding velocities. Continuity equation is then discretised as:

\[ \nabla \cdot \mathbf{U} = \sum_f \mathbf{S} \mathbf{U}_f = 0 \quad (4.11) \]

By substituting Eq. (4.10) into Eq. (4.11) the pressure equation is derived:

\[ \nabla \cdot \left( \frac{1}{A} \nabla p \right) = \nabla \cdot \left( \frac{\mathbf{H(U)}}{A} \right) \quad (4.12) \]
The mass flux through a cell can then be obtained:

\[
\phi = S_f \left[ \left( \frac{H}{A} \right)_f - \left( \frac{1}{A} \right)_f (\nabla p)_f \right]
\] (4.13)

The solution procedure for the SIMPLE algorithm is shown below:

1. Set boundary conditions.
2. Solve discretised momentum equations and obtain the intermediate velocity field.
3. Compute uncorrected mass fluxes at cell faces.
4. Solve pressure correction equation and apply under-relaxation.
5. Correct mass fluxes.
6. Correct cell velocities.
7. Update boundary conditions.
8. Repeat till convergence.

4.4 Wall Functions in OpenFOAM

All wall functions are boundary conditions and in OpenFOAM inherit from the abstract class `FvPatchField`, which can either be a Dirichlet or a Neumann boundary condition. Their definition exist in the functions `updateCoeffs` or `calculate`.

The key parameters of \( k - \varepsilon \) turbulence model include \( k, \varepsilon \) and \( \nu_\text{t} \), for each of them OpenFOAM includes a wall function, that calculates the value for the first cell as shown below:

- **kQRWallFunction** inherits from the `zeroGradientFvPatchField`, which means it provides a Neumann boundary condition, i.e pure zero-gradient boundary condition and it is used for High-Reynolds number cases.
• **epsilonWallFunctions** calculates the $\varepsilon$ in the first cell center and constrains it to that value. Moreover, it is used for the calculation of the production term of the turbulent kinetic energy ($P$, or $G$ in OpenFOAM), which is then inserted into the $k$ equation.

• **nutWallFunctions**, which has different implementations in OpenFOAM, in this study the nutkWallFunction will be discussed. Purpose of the nutkWallFunction is to calculate turbulent viscosity on the wall in order to obtain the correct velocity at the first cell, based on the *law of the wall*.

### 4.4.1 Generalised Wall Function Implementation

A major difference of GWF, in contrast with SWF, as proposed by Popovac and Hanjlic is the need to express the velocity field, as well as its gradients in the local wall coordinate system, as can be seen in Fig. 4.2:

![Wall based coordinate system](image)

**Figure 4.2**: Wall based coordinate system

The implantation work-flow is described below:

1. The velocity field is expressed in the wall coordinate system.

2. The gradient of the velocity and pressure are also transformed into wall tangential and wall normal components.

3. The term $C_U$, which represent the non-equilibrium effects, is computed from Eq. (3.20).
4. $C_U$ is non-dimensionalised as $C_U^* = C_U \nu / (u_k^2 \sqrt{\kappa})$.

5. The non equilibrium function $\psi$ is obtained from $\psi = 1 - C_U^* y^*/(\kappa^* U^*)$.

6. Turbulence viscosity is then computed, using CWT as described in the previous chapter, as $\nu_t = \nu(y^* \kappa^* \psi / \ln (E^*y^*) - 1)e^{-1/\Gamma}$.

7. Finally, the production of turbulent kinetic energy and the dissipation rate are obtained through Eq. (3.33) and Eq. (3.34) respectively.

An important note during the implementation of GWF, is to limit the value of the non equilibrium function $\psi$ not to fall below zero as that will lead to nonphysical values for all the variables of interest.
Chapter 5

Validation Studies

After the Generalised Wall Functions together with the Compound Wall Treatment have been implemented in the $k-\varepsilon$ turbulence model of OpenFOAM, they must be validated with experimental data for different cases. The three selected cases were the Asymmetric Plane Diffuser, Wall Mounted Hump and the 2D-NACA 4412, which together represent a variety of flow phenomena like strong pressure gradients, separation and reattachment; proving, this way, the success of the new wall treatments methods in such conditions.

For all cases, two grids are used: a dense grid ($0 < y^+ < 5$) to test the Low-Reynolds approach and a coarser grid ($5 < y^+ < 100$) to test the Wall Functions. The steady state approach with the SIMPLE algorithm was selected, as the cases do not show severe unsteady phenomena. The linear-upwind scheme is used for the discretisation of the convection of velocity, while for the rest of the convective terms, the first order upwind scheme is used. Gaussian integration is used for both the gradients and laplacian terms with linear interpolation to obtain face values. The algebraic multigrid solver was selected for the solution of the pressure equation, while all others are solved with the smooth solver with a Gauss-Seidel smoother.
5.1 Asymmetric Plane Diffuser

The Asymmetric Plane Diffuser has been studied at least by two different research groups: Obi et al at Keio University in Japan, and Buice & Eaton at Stanford University. It has received a lot of attention due to its unique features, essential for developing turbulence models and wall functions, such as: the fully-developed inlet conditions, separation and reattachment and finally redevelopment of the downstream boundary layer.

5.1.1 Case description

The geometry used for this study is obtained from Obi and can be seen in Fig. 5.1. The diffuser has an opening angle of $\alpha = 10^\circ$ and an expansion ratio of 4.7. The inlet flow is two-dimensional, incompressible, turbulent and fully-developed channel flow with a Reynolds number of 20,000 based on the centerline velocity and channel height.

Two different grids were developed for this study, as posed in Fig. 5.2. The left corresponds to a $y^+ \approx 60$ and was used for the High-Reynolds cases while the right corresponds to a $y^+ \approx 0.1$ for the Low-Reynolds cases.

5.1.2 Results

Fig. 5.3 and Fig. 5.4 show the qualitative comparison of velocity profiles and pressure coefficient through the diffuser geometry between HRN and LRN approaches. It is evident that the Generalised Wall Functions has significantly better results than the Standard Wall Functions,
Chapter 5. Validation Studies

Figure 5.2: Computational grids for the Asymmetric Plane Diffuser case

especially inside the separation zone. Problems may occur in regions where the wall tangential
velocity changes directions at $x/h \approx 9$ and $x/h \approx 35$, as will be discussed in the following
sections.

Figure 5.3: Comparison of velocity profiles through diffuser for Standard Wall Functions, Generalised Wall Functions results and experimental data.

The pressure coefficients results suggest that the GWF matches better the experimental data
even from the LRN model, something that highlights the erroneous behavior of the $k-\varepsilon$ model
close to the wall even with extremely fine grids.
5.2 Wall Mounted Hump

5.2.1 Case Description

Primary focus of this case is to assess the ability of turbulence models and consequently of wall functions to predict low speed separation from a smooth body, due to adverse pressure gradient, as well as reattachment and boundary layer recovery.

The geometry is a Wall-mounted Glauert-Goldschmied type body, geometrically similar to that employed by Seifert & Pack (2002), as posed in Fig. 5.5. The length of the hump is $c = 420\text{mm}$ and its maximum height is $h = 53.7\text{mm}$. A contoured top wall is used in the simulations to account for side-endplate effects.

The flow was nominally 2D with freestream velocity approximately $34.6\text{ m/s (M = 0.1)}$ and Reynolds number based on the length of the hump $Re_c = 936000$.

The same discretisation procedure and solution methods was used as the Assymetric Plane Dif-
fuser case. The grid for the Low Reynolds number case was obtained from NASA’s Turbulence Modeling Resource site with $y^+$ value below 0.1. For the High Reynolds number case a grid was created with $y^+ \approx 80$. The grids are presented below:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{comparison.png}
\caption{Comparison of pressure coefficients for Low Reynolds $k - \varepsilon$, Standard Wall Functions, Generalised Wall Functions results and experimental data.}
\end{figure}
5.2.2 Results

The results shown in Fig. 5.7 and Fig. 5.8 again validate the argument that the Generalised Wall Functions improve the near-wall predictions even in the presence of severe non equilibrium effects as seen in the Wall-mounted hump case. In particular, the velocity profile at $x/c = 0.64$ for the GWF matches better the experimental data, meaning there is an accurate prediction across the whole favorable pressure gradient area upstream of it. Inside the separation zone the GWF shows better results, especially in the freestream values, due to the better estimation of the upstream flow. Further downstream where the reattachment begins and the pressure gradient has significant effect, non equilibrium correction helps GWF accelerate and reattach.
faster than the SWF.

Figure 5.7: Comparison of velocity profiles for Standard Wall Functions, Generalised Wall Functions results and experimental data.

The pressure variation until the separation point is basically the same for all the cases and closely matches the experimental values. However, at the separation zone the SWF over-predicts the pressure coefficient while it fails to adapt to the sudden pressure rise. GWF shows more promising results in terms of pressure coefficient values, but it also lags at the pressure recovery. Finally, Low Reynolds $k-\varepsilon$ model shows a good agreement with experimental values, except from the slight under-prediction inside the separation bubble.

5.3 2D-NACA 4412

5.3.1 Case Description

The Generalised Wall Functions were also validated with the test case of flow over NACA4412 airfoil as described in NASA’s *Turbulence Modelling Resource* webpage. The experimental data was collected by Wadcock and Coles in the GALCIT (Graduate Aeronautical Laboratories, California Institute of Technology) 10-ft wind tunnel.

The airfoil was set at an angle of 13.87 degrees and the inlet velocity was measured at 31.13 $m/s$ ($M =$
Chapter 5. Validation Studies

Figure 5.8: Comparison of pressure coefficients for Low Reynolds $k - \varepsilon$, Standard Wall Functions, Generalised Wall Functions results and experimental data.

0.09), leading to a Reynolds number per airfoil chord of 1.52 million. The computational grids, both for the Low Reynolds number and High Reynolds number approaches, had a farfield outer boundary extending to 500c, which has proved to give better drag coefficient results.

Figure 5.9: Computational grids for the 2D-N4412 airfoil case.
5.3.2 Results

The results for the lift and drag coefficients are summarised in the following table, for the Standard Wall Functions, Generalised Wall Functions, Low-Re $k - \varepsilon$ model and experimental data:

<table>
<thead>
<tr>
<th></th>
<th>LRN</th>
<th>SWF</th>
<th>GWF</th>
<th>Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift Coefficient $C_l$</td>
<td>1.69</td>
<td>1.38</td>
<td>1.71</td>
<td>1.64</td>
</tr>
<tr>
<td>Drag Coefficient $C_d$</td>
<td>0.029</td>
<td>0.058</td>
<td>0.026</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Table 5.1: Lift and Drag coefficients for the NACA4412 airfoil.

In the SWF there is a large separation zone in the suction side of the airfoil, as it is demonstrated by the pressure coefficient variation in Fig. 5.10 where there is a large plateau after the middle of the upper surface. This leads to extreme under-prediction of the lift coefficient and over-prediction of the drag-coefficient due to separation.

On the other hand the GWF show a completely different picture from the SWF. The pressure coefficient closely matches that from the LRN model and the experimental data, until the trailing part of airfoil, where it fails to predict the small separation zone that is formed.
Figure 5.11: Comparison of velocity profiles for Low Reynolds $k - \varepsilon$, Standard Wall Functions, Generalised Wall Functions results and experimental data, across the trailing part of the airfoil.

In Fig. 5.11 the velocity profiles in the trailing edge of the airfoil highlight the previous observations. Specifically, until $x/c = 0.78$ both the GWF and LRN agree with the experimental results, in contrast SWF already at $x/c = 0.73$ the flow starts to separate and reverses. Further downstream where the actual separation begins the LRN fails to predict it, while the correction of the non equilibrium effects in GWF brings the velocity distribution close the LRN and not
5.4 Non equilibrium terms

In Fig. 5.12 the development of the non equilibrium terms is presented for all three test-cases:

The first observation that is easily conducted is the dominance of the pressure gradient term in $C_U$, which means the non equilibrium function $\psi$ will correct the variables of interest mainly based on the pressure gradient variation. Apart for some regions where the velocity gradient normal to the wall takes significant values, in the majority of the near wall cells, the normal and tangential velocity gradients cancel each-other out.

In the Wall Mounted Hump case, there are some tiny oscillations of the $vdU/dy$ term in the
favorable pressure gradient region, which then cause $C_U$ also to oscillate. This happens due to
the normal velocity component being close to zero and thus changing signs rapidly, even if the
velocity component is practically zero the multiplication with the velocity gradient magnifies
the effect. However, this has not a significant effect in the overall performance of the GWF.

In all three cases an interesting phenomena occurred when the velocity starts to reverse direction
and its magnitude tends to zero. From Eq. (3.26) when $U_P \to 0 \Rightarrow \psi \to \pm \infty$ depending on the
sign of $C_U$. This behavior is clearly shown in Fig. 5.13a where the values for the non equilibrium
function across the Wall-Mounted Hump are plotted. At $x/c \approx 0.7$ $\psi$ reaches a value above 14,
while the negative values have been filtered out as described in Section 4.4. Same behavior is
also present in the other two cases, as far as the writer knows this does not have been proved
to lead to errors in the simulation.
If the velocity magnitude in the non equilibrium equation $\psi$ is non dimensionalised with the friction velocity instead of $u_k$, then the sudden peaks are smoothed out and $\psi$ values are distributed around the equilibrium value of 1, as seen in Fig. 5.13b, although this may lead to smaller corrections in the velocity profiles. Further investigation is needed in order to produce meaningful results.

5.5 Closure

Ending this chapter a comparison between the simulation time for the different near wall models is posed in Table 5.2. It is evident that the use of wall functions increases dramatically the speed of simulations, without taking into account also the human hours needed for the grid generation of either approach, which will, of course, be in favor of the easily generated High Reynolds number meshes.

<table>
<thead>
<tr>
<th></th>
<th>Assymetric Diffuser</th>
<th>Wall-Mounted Hump</th>
<th>NACA4412</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LRN</td>
<td>SWF</td>
<td>GWF</td>
</tr>
<tr>
<td>No. of cells</td>
<td>1 8.3 8.3</td>
<td>1 1.4 1.4</td>
<td>1 1.5 1.5</td>
</tr>
<tr>
<td>Clock Time</td>
<td>1 102 95</td>
<td>1 3.7 3.3</td>
<td>1 2 1.9</td>
</tr>
</tbody>
</table>

Table 5.2: Number of cells and Clock time non-dimensionalised with the Low-Reynolds $k - \varepsilon$ model for Standard and Generalised Wall Functions approaches.
Chapter 6

Conclusions and Recommendations

6.1 Conclusions

Several wall treatments methods for the two equation model, $k - \varepsilon$, are presented, discussed and implemented in OpenFOAM. Three test cases were used for validation, representing flows in non-equilibrium, separation and reattachment. The main findings with respect to these simulations are:

- The $k - \varepsilon$ model struggles to accurately represent near-wall phenomena.
- Standard Wall Function approach fails in regions where pressure gradients effects dominate the flow, due to the equilibrium assumptions.
- Generalised Wall Function is the most easy to implement wall treatment method, incorporating non-equilibrium effects.
- With the use of the Compound Wall Treatment method the first cell height becomes irrelevant, as the correct behavior is returned, whether it lies in the sublayer, buffer layer or the logarithmic layer.
- The combined use of GWF with CWT proved to give the best results compared with the standard methods in all three cases. The non equilibrium equation helps correcting the
velocity profiles and matching those from the Low Reynolds model and the experimental data, at a fraction of time.

6.2 Future Work

During this work, a useful set of results were produced regarding the generalisation of wall treatment methods. However, much more work can be done in order to provide a complete picture for these methods. Part of the future work may include:

- Incorporate the Generalised Wall Function and Compound Wall Treatment in other turbulence models that have significant better behavior in wall bounded flows.

- Validate GWF and CWT in more cases, including three-dimensional effects and heat transfer.

- A different non-dimensionalisation of the non equilibrium function may lead to better results in regions where the velocity magnitude tends to zero.

- Further investigation of the development of non equilibrium terms and their impact on the non equilibrium function $\psi$ is needed.
References


of hydrodynamic circulating flow”. In: *Applied Mathematical Modelling* (2013).


[7] B. Launder and D. Spalding. “The numerical computation of turbulent flows”. In:

for turbulent convective heat transfer”. In: *9th UK National Heat Transfer Conference,


Appendix A

nutkGWF

The code for the implementation of Generalised Wall Function and Compound Wall Treatment in nutkWallFunction is presented below:

```cpp
tmp<scalarField> nutkGWFFvPatchScalarField::calcNut() const
{
    const label patchi = patch().index();

    const turbulenceModel& turbModel = db().lookupObject<turbulenceModel>(
        IOobject::groupName(turbulenceModel::propertiesName, internalField().group()));

    const fvPatchVectorField& Uw = turbModel.U().boundaryField()[patchi];
    const scalarField magUp(mag(Uw.patchInternalField() - Uw));
    const scalarField magGradUw(mag(Uw.snGrad()));
    const scalarField& y = turbModel.y()[patchi];
    const tmp<volScalarField> tk = turbModel.k();
    const volScalarField& k = tk();
```
const tmp<scalarField> tnuw = turbModel.nu(patchi);
const scalarField& nuw = tnuw();
const scalar Cmu25 = pow025(Cmu_);
tmp<scalarField> tnutw(new scalarField(patch().size(), 0.0));
scalarField& nutw = tnutw.ref();

// Generalised Wall Function
volVectorField vU = turbModel.U(); // take Velocity Field
const labelList cells = patch().faceCells();
tmp<vectorField> tnv = patch().Sf()/patch().magSf(); //normal face vector
vectorField& nv = tnv.ref();
vectorField tv = nv; //init tangential vector
scalarField alpha = nutw; //init alpha, angle between coordinate systems
scalarField beta = nutw;
scalarField gamma = nutw;
scalarField Ut = nutw;
scalarField Un = nutw;

forAll(nv,celli)
{
    if (nv[celli].component(2) < 0)
    {
        tv[celli].component(0) = -nv[celli].component(2);
        tv[celli].component(2) = nv[celli].component(0);
    }

    if (nv[celli].component(2) >= 0)
    {
        tv[celli].component(0) = nv[celli].component(2);
        tv[celli].component(2) = -nv[celli].component(0);
    }
\[
\alpha_{celli} = \arctan2(tv_{celli}.\text{component}(2),tv_{celli}.\text{component}(0));
\]
\[
\beta_{celli} = \arctan2(vU_{celli}.\text{component}(2),vU_{celli}.\text{component}(0));
\]
\[
\gamma_{celli} = \beta_{celli} - \alpha_{celli};
\]
\[
U_t_{celli} = \text{magUp}_{celli} \times \cos(\gamma_{celli}); \quad \text{Wall Velocity Component}
\]
\[
U_n_{celli} = \text{magUp}_{celli} \times \sin(\gamma_{celli}); \quad \text{Normal Wall Velocity Component}
\]

```cpp
alpha[celli] = atan2(tv[celli].component(2),tv[celli].component(0));
beta[celli] = atan2(vU[cells[celli]].component(2),(vU[cells[celli]].component(0))); // angle between velocity and x-axis
gamma[celli] = beta[celli] - alpha[celli]; // angle between velocity and wall
Ut[celli] = magUp[celli] * cos(gamma[celli]); // Wall Velocity Component
Un[celli] = magUp[celli] * sin(gamma[celli]); // Normal Wall Velocity Component
```
dPdt[celli] = gradP[celli] & tv[celli]; // dP / dw

}

forAll(nutw, facei)
{
    label celli = patch().faceCells()[facei];
    scalar yPlus = Cmu25*y[facei]*sqrt(k[celli])/nuw[facei];

    // Non dimensionless Psi Calculation
    scalar kstar = kappa_*Cmu25;
    scalar uk = Cmu25*sqrt(k[celli]);
    scalar utau = sqrt(nuw[facei]*mag(dUttn[facei]))+1.0e-12;
    scalar Ustar = mag(Ut[facei]) /uk;

    scalar Custar = Cu[facei] * nuw[facei] / (sqr(uk)*sqrt(k[celli]));
    scalar psi = 1 - Custar * yPlus / (kstar * Ustar);

    if (psi <= 0.1)
    {
        psi = 0.1;
    }

    // Compound Wall Treatment
    scalar Gamma = 0.01 * pow(yPlus,4) / (1 + 5 * yPlus);
    nutw[facei] = nuw[facei]*(yPlus*kappa_*psi/log(E_*yPlus) - 1.0)*Foam::exp(-1/Gamma);
}

return tnutw;
Appendix B

epsilonGWF

The code for the implementation of Generalised Wall Function and Compound Wall Treatment in epsilonWallFunction is presented below, without the transformation of coordinates, which has been mentioned in Appendix A:

```plaintext
forAll(nuw, facei)
{
    const label celli = patch.faceCells()[facei];
    const scalar yPlus = Cmu25*y[facei]*sqrt(k[celli])/nuw[facei];
    const scalar w = cornerWeights[facei];

    scalar kstar = kappa_*Cmu25;
    scalar uk = Cmu25*sqrt(k[celli]);
    scalar utau = sqrt(nuw[facei]*mag(dUtdn[facei]))+1e-12;
    scalar Ustar = mag(Ut[facei]) /uk;
    scalar Custar = Cu[facei] * nuw[facei] / (sqr(uk)*sqrt(k[celli]));
    scalar psi = 1 - Custar * yPlus / (kstar * Ustar );

    if (psi <= 0.1 )
```

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{  
    psi = 0.1;  
}

// Compound Wall Treatment

calar Gamma = 0.01 * pow(yPlus,4) / (1 + 5 * yPlus);
calar Gamma2 = 0.001 * pow(yPlus,4) / (1 + yPlus);

epsilon0[celli] += w*(2.0*k[celli]*nuw[facei]/sqr(y[facei]) * Foam::exp(-Gamma2)
  - Cmu75*pow(k[celli], 1.5)/(psi*kappa_*y[facei]) * Foam::exp(-1/Gamma2));

G0[celli] +=
  w*(Cmu_*sqr(k[celli])/(epsilon0[celli]*sqr(magGradUw[facei]))*Foam::exp(-Gamma)
  + (nutw[facei] +
  nuw[facei])*magGradUw[facei]*Cmu25*sqrt(k[celli])/(kappa_*y[facei]) * 
  Foam::exp(-1/Gamma));
}