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**Citation:**
FRAC TAL-BASED APPROACH FOR MODELING ELECTRIC BREAKDOWN OF AIR GAPS: AN APPLICATION TO A 75 CM POSITIVE ROD-PLANE GAP

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Abstract: A stochastic model for the dynamic simulation of the spatial growth of electrical discharges as well as for assessing breakdown of air gaps is presented. The fractal-based model visualizes the stochastic behavior of discharge development, in terms of channel branching and tortuosity, and yields a statistical distribution for the dielectric strength of the gap as affected by discharge evolution. The main algorithm of the proposed simulation model, developed in MATLAB environment, is applicable in both two (2D) and three (3D) dimensions and considers as influencing parameters on discharge evolution: the critical field required for discharge evolution, the voltage drop along the discharge channel, the propagation parameter “η”, commonly employed in fractal studies, to account for the dependence of propagation probability on the electric field, as well as the size of the mesh. The electric potential in the gap is calculated at each step of discharge evolution by using the finite difference method (FDM) with a successive over relaxation (SOR) technique. Simulation is based on an iterative computation until either no point in space satisfies the evolution criterion (withstand case) or when the electrical discharge bridges the gap (breakdown case). Discharge evolution is simulated dynamically, by recalculating the electric potential at each growth step; thus, the path of the discharge is computed as the latter evolves under the interaction of all discharge branches. An application of the model is made to obtain the breakdown probability distribution of a 75 cm positive rod-plane air gap. By selecting appropriate values for the model key parameters, the stochastic behavior of discharge development in terms of branching and tortuosity is visualized. 2D and 3D simulations are conducted adopting the “multiple-level tests” method according to IEC 60060-1, so as to compute the 50% breakdown voltage and standard deviation. Simulation results are evaluated and discussed with reference to the experimentally obtained breakdown probability distribution; a very good agreement is shown to exist. Values of the model key parameters are discussed on the basis of 2D and 3D simulations.

1 INTRODUCTION

Electrical discharges in gaseous, solid and liquid dielectrics are strongly characterized by features of branching and tortuosity, both in prebreakdown and breakdown phenomena. Since 1982, when B. Mandelbrot [1], firstly introduced the fractal geometry many researchers have employed the concept of fractals and their main characteristic of self-similarity, in order to simulate the stochastic behavior of electrical discharge evolution [2-5].

Various stochastic models were introduced to simulate the dynamic growth of electrical discharges and to describe these complex patterns taking into account their fractal characteristics. They were implemented to a number of applications such as simulation of surface discharges evolution (Lichtenberg figures) [2, 6], breakdown and pre-breakdown phenomena in dielectrics [3, 7-9] as well as lightning protection [10-12]. The major asset of all these models is their ability to combine both the deterministic and statistical features of electrical discharges so as to consider the stochastic nature of dielectric breakdown and derive more accurate results on discharge evolution.

In this study a stochastic model for the simulation of the spatial growth of electrical discharges as well as for obtaining breakdown probability distribution of air gaps is presented. The discharge evolution is simulated through steps of development, by calculating at each step the electric potential in space. The fractal-based model visualizes the stochastic behavior of discharge development in terms of channel branching and tortuosity and yields a statistical distribution for the dielectric strength of the gap as affected by the boundary conditions. An application of the fractal-based model is made for a 75 cm positive rod-plane air gap. Simulation results are evaluated and discussed with reference to the experimentally obtained breakdown probability distribution under
positive lightning impulse voltages. Values of key parameters of the model are discussed based on physical interpretations of simulation results considering both the 2D and 3D approaches.

2 MODEL DESCRIPTION

2.1 MAIN ALGORITHM

The main algorithm of the proposed stochastic model has been developed in the MATLAB environment. The space adopted in the simulations is discretized in both two and three dimensions using an appropriate mesh size. The electrical discharge emanates from the high voltage electrode, and progresses towards the grounded electrode. Downward discharge evolves in steps, and at each step, which is equal to mesh size, the electric potential, \( V \), is calculated at the whole space. Firstly, the average potential gradient between all nodes \( P \) that are already part of the discharge channel and their surrounding points \( P' \) are calculated from the equation:

\[
E_{pp} = \frac{|V_p - V_{p'}|}{D_{pp}}
\]  

(1)

where \( V_p \) and \( V_{p'} \) are the electric potentials of points \( P \) and \( P' \), respectively and \( D_{pp} \) is the distance between \( P \) and \( P' \) (Figure 1).

Possible candidate points of discharge progression are only those that fulfil the following propagation criterion:

\[
E_{pp} \geq E_{cr}
\]

(2)

where \( E_{cr} \) is a critical electric field value required for discharge evolution.

For example, considering that \( P \) is a point of the discharge then up to 8 surrounding points in 2D (Figure 1a) and up to 26 surrounding points in 3D (Figure 1b), including diagonal ones, are possible candidate points, \( M \). Thus, the total candidate points at each step are \( M < 8N \) (Figure 2, 2D) or \( < 26N \) (3D), where \( N \) is the number of the discharge path nodes.

\[\begin{align*}
E_{pp} & = \frac{|V_p - V_{p'}|}{D_{pp}} \\
\end{align*}\]

\[\begin{align*}
E_{pp} & \geq E_{cr}
\end{align*}\]

The propagation probability \( p(P \rightarrow P') \) of a surrounding point \( P' \) to become part of the electrical discharge channel is calculated by [12]:

\[
p(P \rightarrow P') = \begin{cases}
\frac{(E_{pp})^\eta}{\sum_i \sum_j (E_{pp_i})^\eta}, & \text{for } E_{pp} \geq E_{cr} \\
0, & \text{for } E_{pp} \leq E_{cr}
\end{cases}
\]

(3)

where \( \eta \) is a propagation parameter [2]. This parameter accounts for the dependence of propagation probability on the electric field. Lower values of \( \eta \) yield more intense branching and tortuosity for the discharge channel, enhancing the stochastic behavior in discharge growth.

The denominator in equation (3) refers to the summation of all possible candidate discharge points \( M \) from all discharge nodes \( N \) and as a result normalizes the propagation probability distribution [13]. A number uniformly distributed in the interval \((0, 1)\) is randomly obtained and by using the cumulative distribution function of \( p(P \rightarrow P') \) (equation 3) the next discharge channel point \( P' \) is chosen [14]. The model belongs to single element category; only one candidate point is selected per step. Then, the electric potential \( V \) of the newly added to the discharge channel point is updated using the formula:

\[
V_p = V_o - s \cdot E_{ch}
\]

(4)

where \( V_o \) is the electric potential at the point of origin of the discharge, \( E_{ch} \) is an average electric field accounting for the voltage drop along the discharge channel, and \( s \) is the length of the discharge along its path, between the point of origin and the newly added discharge point.

This procedure continues iteratively until either no point satisfies the propagation criterion (equation 2) (withstand case) or when the electrical discharge channel reaches the grounded electrode (breakdown case).

Figure 1: Full dot (●) denotes a discharge node, \( P \), and empty dots (○) the surrounding points, \( P' \), of the node; a) 2D and b) 3D.

Figure 2: Discharge points for a 2D simulation scenario. Full dots (●) denote the discharge nodes, empty dots (○) the candidate points and crosses (×) points that do not fulfil the propagation criterion.
2.2 ELECTRIC POTENTIAL CALCULATION

Electric potential calculation is the most demanding part in the stochastic model, leading to huge computational cost. In the present model, the finite difference method (FDM) is used in both two and three dimensions in order to solve Laplace’s equation and calculate the electric potential $V$ at every point in the simulation area.

$$\nabla^2 V = 0 \quad (5)$$

From the theory of partial differential equations (PDE’s), in a computational domain discretized using equal grid spacings in all directions and neglecting higher order terms in Taylor’s expansion, Laplace’s equation (5) is transformed into the below discretized form in 2 dimensions

$$V_{i,j} = \frac{1}{4} \left( V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1} \right) \quad (6)$$

which is known as the “5-point approximation” and to the “7-point approximation” in 3 dimensions respectively [15].

$$V_{i,j,k} = \frac{\left( V_{i+1,j,k} + V_{i-1,j,k} + V_{i,j+1,k} + V_{i,j-1,k} + V_{i,j,k+1} + V_{i,j,k-1} \right)}{6} \quad (7)$$

In order to solve the above equations, a successive-over relaxation (SOR) technique is used, which is found to significantly reduce the calculation time and reach faster to convergence than other iterative methods [15, 16].

$$V_{i,j}^{n+1} = (1 - \omega) \cdot V_{i,j}^n + \omega \cdot V_{i,j}^* \quad (8)$$

where

$$V_{i,j}^* = \frac{1}{4} \left( V_{i+1,j}^{n+1} + V_{i-1,j}^{n+1} + V_{i,j+1}^{n+1} + V_{i,j-1}^{n+1} \right) \quad (9)$$

and $V_{i,j}^{n+1}$ denotes the $(n+1)^{th}$ estimate of the potential solution [16]. Equations (8) and (9) can be derived for 3 dimensions after appropriate adjustments. The over-relaxation parameter, $\omega$, takes values between 1 and 2 and the optimal value speeds up the simulation process [15, 16]. In this study the values of 1.9 and 1.6 [17] were used for 2D and 3D simulations, respectively.

The electric potential is calculated by iteratively solving equation (8) until the relative error in the potential for all points, between two successive iterations $n$ and $n+1$ of the SOR algorithm, is smaller than 1%. Two types of boundary conditions were applied. Dirichlet boundary conditions ($V=\text{ct.}$) were imposed on objects with fixed potential (electrodes), as well as on discharge nodes. Neumann type boundary conditions ($dV/dx = 0$), were chosen for the lateral boundaries defining the simulation area. When a new point is added to the discharge channel, its potential is updated according to equation (4) and then, with the new boundary conditions imposed, the potential in the whole space is recalculated. Consequently, the modification of the potential at every new discharge point affects the whole simulation and confers on it a dynamic attribute.

3 MODEL APPLICATION

An application of the fractal-based model is presented for the case of a rod-plane air gap 75 cm in length. Under positive applied voltages, breakdown probability curves were obtained by performing 2D and 3D simulations. The key parameter values shown in Table 1 were employed in simulations, yielding satisfactory agreement with experimental data and reasonable simulation time. For the average electric field accounting for the voltage drop along the discharge channel, $E_{ch}$, a value of 4.5 kV/cm has been used, in accordance with the threshold positive streamer propagation field [18]. The mesh size of 1 cm was considered adequate given the length of the gap and computing effort; it must be noted that $E_{cr}$ and $\eta$ may vary depending on mesh size.

<table>
<thead>
<tr>
<th>Table 1: Key parameter values</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$E_{ch}$ (kV/cm)</td>
</tr>
<tr>
<td>$E_{cr}$ (kV/cm)</td>
</tr>
<tr>
<td>$\eta$</td>
</tr>
<tr>
<td>Mesh size (cm)</td>
</tr>
</tbody>
</table>

3.1 SIMULATION RESULTS

Figure 3 shows the discharge evolution for the case of 400 kV applied voltage at the rod; it depicts both breakdown (Figure 3a) and withstand (Figure 3b) cases by visualizing the stochastic behavior of discharge development in terms of channel branching and tortuosity. It should be noted that as the discharge evolution is simulated dynamically, the electric potential is recalculated at each step taking into account the updated value of electric potential of the newly added point, thus the discharge path is computed under the interaction of all discharge branches.

As an illustrative example, Figure 4 shows the increase of the electric potential at 1 cm above the grounded plane at gap axis as the downward discharge develops (simulation steps); it is evident that 2D simulations, although significantly faster, are associated with overestimated values of electric potential in the gap.
Figure 3: Discharge growth under 400 kV applied voltage; 75 cm rod-plane gap, values in cm a) breakdown, b) withstand.

Figure 4: Variation of the electric potential at 1 cm above the grounded plane at gap axis as the downward discharge propagates; 2D (93 steps) and 3D (1040 steps). 400 kV applied voltage causing breakdown.

Figure 5: Variation of the electric potential at 1 cm above the grounded plane at gap axis as the downward discharge propagates. Comparison between a breakdown and a withstand case. 400 kV applied voltage, 3D simulations.

In order to obtain the breakdown probability distribution of the investigated gap, 2D and 3D simulations were performed by adopting the “multiple-level tests” method according to IEC [19]. 100 simulations (voltage applications) per voltage level were run at gradually increasing voltage levels. Figure 6 shows simulation results together with the corresponding breakdown probability distribution obtained experimentally [20]. The probability distributions, were found to be well approximated with the Normal distribution; thus, from each distribution the 50% breakdown voltage, \( U_{50} \), and the associated standard deviation, \( \sigma \), were computed. These values, together with those obtained experimentally, are listed in Table 2.

As evident from Figure 6 and Table 2, there is an excellent agreement between simulation and experimental results. It must be noted that the \( U_{50} \) values of Table 2 compare well with the \( U_{50} \) value of 397.5 kV obtained using the empirical expression for lightning impulse breakdown voltage

\[
U_{50} = 530 \cdot d \quad [21]
\]  

Figure 6: Breakdown probability distributions of the 75 cm rod-plane air gap under positive applied voltages; fitting curves, were drawn according to Normal distribution.
Table 2: U50 and σ of 75 cm positive rod-plane gap

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2D</th>
<th>3D</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U50 (kV)</strong></td>
<td>395.2</td>
<td>395.7</td>
<td>395.9</td>
</tr>
<tr>
<td><strong>σ (%)</strong></td>
<td>1.5</td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>Total time</strong></td>
<td>1 hour (100 voltage appl. per voltage level)</td>
<td>52 hours (100 voltage appl. per voltage level)</td>
<td>2 hours (20 voltage appl. per voltage level)</td>
</tr>
<tr>
<td><strong>Intel Core i7-8700, 3.2 GHz, 32GB RAM</strong></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

and they are also higher (~13%) than that of the DC breakdown voltage of the investigated gap [21].

3.2 DISCUSSION

Rod-plane air gaps are axis-symmetrical configurations allowing for electrostatic field and electrical potential computation in 2 dimensions. However, where in simulations the growth of electrical discharges is of concern, the symmetry of the gap does not apply. Actually, the more intense the branching and tortuosity of the discharge channel the more asymmetric becomes the electric field distribution between the electrodes. Thus, 3D simulations are more appropriate for calculating accurately the electric field and potential distributions.

Nevertheless, because of the significantly lower computational cost related to 2D simulations (Table 2), the latter may also be employed in engineering applications by considering a planar approach; however, this requires appropriate adjustment of key parameters values in simulations for compensating the overestimation of the electric potential within the gap (Figure 4) as well as the fact that discharge growth is limited in 2 dimensions. Actually, using the same mesh size (1 cm) and average gradient of positive discharge (Ecr=4.5 kV/cm), to obtain a satisfactory agreement between simulation and experimental results, lower values for the critical electric field for discharge evolution, Ecr in equation (2), and propagation parameter, η in equation (3), had to be employed in 2D than 3D simulations. It is important to note that 3D simulation results, as compared to 2D ones, (U50, σ, tortuosity, branching) are more sensitive to η variations, due to the significantly larger number of candidate points for discharge evolution.

Future work necessitates the integration of time to the fractal-based model, so as to take into account the temporal variation of discharge evolution under voltages both stable and variable in time. Also, a sensitivity analysis on the dependence of the model key parameters on mesh size, gap length and electrode configuration is currently undertaken; this, together with relevant experimental results, may allow for the physical interpretation of the model parameters and simulation results.

4 CONCLUSIONS

A stochastic model for the simulation of electrical discharge spatial evolution and for determining the dielectric strength of non-uniform air gaps has been introduced. Discharge evolution is simulated dynamically, by recalculating the electric potential at each growth step; thus, the path of the discharge is computed as the latter evolves under the interaction of all discharge branches.

The fractal-based model has been applied to obtain the breakdown probability distribution of a 75 cm rod-plane air gap under positive lightning impulse voltages. By selecting appropriate values for the model key parameters, the stochastic behavior of discharge development in terms of channel branching and tortuosity has been visualized. The computed 50% breakdown voltage and standard deviation σ are in excellent agreement with experimental results.

Accurate simulation of the dynamic growth of electrical discharges in air gaps requires problem solution in 3 dimensions. 2D simulations may yield acceptable results on breakdown probability distribution of air gaps after appropriately adjusting key model parameters, such as the critical electric field required for discharge evolution and propagation parameter, both affecting branching and tortuosity of the discharge channel.

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REFERENCES


