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Dissertation
«Quantitative analysis of Inventory Management System - Case study of an Auto-Parts Warehouse»

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Abstract

Inventory management systems are a vital part of any organization that has to keep order and trace the products that are immediately available and those that have to be ordered or are in the process of being ordered and transported to the firm’s warehouse.

Purpose: The purpose of this thesis is to analyze the inventory management system of an existing company in using real data and propose better ways to forecast demand, place optimal order quantities and minimize the costs of operation.

Method: The literature review used articles of scientific journals, studies and books on the subject from the international literature through the Google Scholar platform and also academic books. Finally, the knowledge and methods gathered from the literature review, were exploited for the interpretation of the results of the analysis and the comparison with the firm’s system.

Results: It’s been proven that the firm has a distance to cover in order of proper utilization of its inventory management system and for the SKU that was taken into consideration it’s been shown that the proposed model is more efficient as it takes into account the peculiarity of the demand data.

Keywords: EOQ, Inventory Management System, Demand forecasting, uncertain demand, Periodic review system, Heuristics
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1 Introduction

Inventory management is an important responsibility for managing a production system. A stock is considered to be the amount of any financial asset, whether material or not, imported into the system and exceeding the quantity of such goods exported by the system. Stocking can either be designed to smooth out the differences between the supply and demand of the goods that can be a result of various factors such as bad planning or other extraordinary phenomena. The necessity of the stock is mainly due to the uncertainty regarding the supply and demand of the goods to meet the needs.

Inventory control is a technique with scientific bases designed to monitor the stored quantities of the goods and make the relevant decisions as when and in what quantity the material should be ordered. An inventory management system is considered to be the set of regulations and controls that determine the amount of inventory, when inventories should be renewed and how large the orders should be.

The main purpose of an inventory management system is to determine first when the goods are to be ordered and second, how large the order should be. Some businesses prefer to maintain long-term relationships with their suppliers to meet their needs for almost a whole year. In this case, an inventory management system will determine when and what quantity will be distributed. An efficient inventory management system saves resources for the business by minimizing the costs.

The concept of inventory is general and is not limited to raw materials, goods and commodities but covers a wide range of economic assets. Regardless of the generality of the term, the problem of inventory management is very important for all companies as their stocks usually bind a large proportion of their capital and have significant maintenance costs. The problem of inventory control has been a great deal of concern in
the literature in recent years and a great effort has been made to analyze and tackle the problem.

In the theoretical approach of the problem, a number of scientific studies have been published, many mathematical analyzes have been made and many inventory management theories and models have been formulated. However, from a practical point of view, only a small part of the theories have been implemented in a real business environment.
2 Literature Review

2.1 Stock and Supply Management

In Logistics Management the dimension of time plays a very important role. The products must be in the right place at the right time and in the right quantity and quality. Because of the time deviation of production and demand, they are bridged by the creation and maintenance of stocks (Sifniotis, 2002). The term inventory refers to any dormant, financial means or resource (product or material) acquired by the enterprise and stores it for use or resale at a future moment. Asset and wealth gathering, as well as stock conservation are a sign of human civilization. However, by the end of 1950, there was a passive attitude of business management towards stockpiles. Since then, this has been overturned, alongside the changing business environment and rapid technological progress (Silver E., 2008).

Inventory management is now an important responsibility for managing a production system. Stocks are mostly deliberately created to smooth the time gaps between supply and demand. In other words, the necessity of stock availability lies in the uncertainty regarding the supply and demand of the goods. Stock control is a technique with scientific bases designed to monitor the quantity of the goods stored and to make the relevant decisions, such as the time and quantity of the order placed on the suppliers and the replenishment of the current stock kept (Stevenson, Hojati, & Cao, 2007).

The problem of inventory management is very important for all businesses as stocks usually bind a large proportion of their capital and have significant maintenance costs. There are categories of businesses such as supermarkets, where their stocks account for about 50% of their assets. They are therefore directly linked to the liquidity
of the company's capital as the decrease in the stock leads to a release of current capital and consequently a reduction in its lending needs (Stevenson, Hojati, & Cao, 2007).

The inventory management problem is generally defined as a problem of balancing between the cost of shortage and the cost of inventory surplus. Proper stock management design disconnects the production system from fluctuations in demand and maintains a smooth flow of production and reduces costs. In particular, forecasting demand, based on historical demand data and other factors such as competition, consumer purchasing power, product life cycle, marketing strategy, etc., is the mainstay of inventory control and management. Also, the desired level of availability of products (a key customer service policy) also affects the amount of stocks held by the firm. Then, depending on whether this height is high or low, the corresponding needs arise in larger or smaller storage spaces respectively (Silver, Pyke, & Peterson, 1998).

The seasonality of supply and demand also make it necessary for a firm to keep stocks. This seasonality relates to the production and supply of certain goods, as is the case with the production of agricultural products, resulting in large price fluctuations which make it beneficial to stock up a large volume of products at the time of harvest (e.g., fruit, cereals, etc.). It also concerns the supply of raw material, as in the case of fish, which are procured in large quantities and at specific times by the processing industries concerned. The strong demand for seasonal products during festive holiday periods, such as Christmas and Easter, forces the company to store more to cope with a periodic increase in demand if that is the case of its product. The same practice should be adopted in the case of implementing promotional actions, which obviously aim at increasing demand. Otherwise, there is a risk of lost sales and defamation of the company due to lack of requested products and failure to meet the customer's expectations for availability (Ehrenthal, Honhon, & Van Woensel, 2014).
Stocks are needed to achieve economies of scale in markets, transport and production. For example, raw materials and/or finished products have a lower unit cost if they are purchased in large quantities. Also, if a stock of finished products is maintained, transportation and construction savings are possible due to higher volume shipments and the production of more product units respectively. However, the production of products or the purchase of large quantities by suppliers presupposes that some products will remain in stock for a significant period of time before they are sold (Anupindi, Chopra, Deshmukh, Van Mieghem, & Zemel, 1999). Therefore, in the new environment of logistics, holistically, the overall cost and expected benefit that emerges throughout the process, from order placement to delivery to customer, must be considered. For example, account should be taken of the compensation between the cost of maintaining stocks and reducing production costs, including the cost of purchasing raw materials from suppliers in the case of production units or the minimization of ordering and holding costs for retailers or wholesalers.

Several factors of uncertainty arise in maintaining stocks, such as the projected rise in price (speculative reasons), the coverage of possible delays of suppliers and more generally when the firm believes there is a risk of a market failure for any reason. Also, in the production process, stocks of semi-finished products and spare parts needed for regular maintenance protect against emergencies (e.g. damage to equipment), thereby reducing operational risks and the possibility of disruption of production. Finally, stock problems in a business may arise as a result of poor forecasts or because product consumption (demand) has fallen sharply for various abstract reasons (Silver E., 2008).

The key questions that any business is called to respond to regarding stock levels are (Stevenson, Hojati, & Cao, 2007):
• What will the business store?
  This concerns the variety ("range") of products managed by a business. A typical example of the dynamic development of this question are the supermarkets which today sell, besides the traditional consumer products, groceries, meat, fish, bakery products, sweets, housewares, etc., utilizing the need of consumers for "one stop shopping" and of course the economies of scale from managing more products without a corresponding increase in the necessary resources.

• In what quantities and for how long? How fast will stock replenishment be made?
  Quantity and time are interrelated concepts, since the desired demand period to be covered, combined with the quantity predicted for that period, determines the stock level, the quantity (i.e. the inventory) that the business plans to acquire to meet its needs. The higher the stock (the higher the stock level), the slower the stock replenishment (it takes a long time between the orders to the supplier), while the smaller the stock (the low stock level), the faster the replenishment of inventory (there is a short time between orders to the supplier).

• At what cost?
  Inventory cost originates from its acquisition cost (purchase-commission), which is agreed between the business and the supplier. Subsequently, its management generates other costs, such as damage costs, depreciation, insurance premiums,

• Which security stocks will the business maintain?
  Security stocks due to demand uncertainty or lateness are important, because in exceptional cases they maintain high levels of customer service and add stocks, increasing the relative costs of maintaining them.

  In conclusion, without proper stocks, serious customer service problems can be created. Stocks are also vital for production operations because their potential shortage may stop or cause a change in production activities. On the other hand, just as shortfalls,
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stock surpluses cause a significant increase in costs. In this context, inventory management is related to decisions that are decisive for the viability of the business, which directly affect both revenue (through the availability of quality products and customer satisfaction) as well as the costs and cash flow of the business.

2.2 Types of Inventory

Inventory, in simple terms, is essentially the quantity that the business (or even the household) has at any given time for the purpose of its future sale (or consumption). Stocks are maintained at all nodes in the supply chain, even at the end of the supply chain, where they are used by consumers almost daily with their purchases being mainly from the various retail outlets (Tersine & Tersine, 1988).

In general, in the supply chain, the following categories of inventories can be found, based on the type of goods (Stevenson, Hojati, & Cao, 2007):

- **Raw materials**: They are used in the production or processing process carried out at the plant (factory, crafts, etc.) for the production of finished or semi-finished (semi-finished) goods. An example in the clothing industry is the fabric, which is led to the cutter to obtain the necessary pre-designed shape of the garment.

- **Work-in-progress**: They are used in the production process to produce finished or semi-finished goods. They differ from raw materials in sense that the work-in-progress goods are complementary to the product or packaging of the product. An example in the apparel industry is the zipper, buttons, etc. which will be placed on the fabric, based on the physical design of the garment.

- **Semi-finished products (stocks in-process)**: They are the semi-finished products that are in production process and need further processing to produce the finished
products. An example in the clothing industry is garments, which are before the final painting stage.

- **Finished products.** They are the ones that are produced at the end of an enterprise's production process. These are the most common business stocks, although they may not be completely ready for disposal to the final customer (e.g. need packaging, etc.). These products can also be brought in the warehouse as is and then be resold by the company, a retailer probably. Broadly, anything that is ready for sale can be categorized as a finished product.

- **Spare parts:** The maintenance of spare parts is related to the type of business and its assets in production equipment for the regular maintenance or handling of unforeseen damage.

- **Consumables:** necessary goods for the business’ day-to-day procedures that are core for the stable functional operation. Products like these are usually considered papers, inks, etc.

It should be noted at this point that there are various cases where a finished product of a particular production process is essentially a raw material for some other process.

In addition, inventories, depending on the way they are created, are distinguished, on account of their origin, in seven categories: cycle, safety, decoupling, seasonal, standby, dead and lastly pipeline stock. These different types of stocks are not distinguished in physical form, but they can be separated as follows (Muchstadt & Sapra, 2010):

- **Cycle Stock.** The cycle inventory is determined directly by the size of the order. The height of the cycle stock depends on the time between two orders. For example, if an order is placed every 1 week, the amount of inventory should be
equal to weekly demand. The longer the time period between two orders, the larger the cycle stock.

- **Safety Stock.** Safety stock is the stock beyond demand forecast to cover emergencies such as unexpected demand growth, supplier delays or unpredictable supply conditions (e.g. strikes, weather, etc.). Future demand forecasts cannot be completely accurate, however, due to its fluctuation and the ever-changing market environment. The safety stock is particularly important in factories (raw material stocks), because production lines cannot work without raw materials, which is a situation that would result in great discrepancies and loss of money.

- **Pipeline Stock.** The pipeline stock or in other words stock in transit or on the move concerns products moving from one faculty to another, such as from the factory to the distribution centers. They are considered part of the cycle inventory although they are not available for sale or dispatch unless they reach their destination.

- **Standby Stock.** The standby stock or standby inventory is used to absorb dissimilar demand over different time periods. For example, 90% of the annual demand for heating oil occurs within four months of the year. By maintaining standby stocks, companies are not obliged to make significant cost reductions in production. Standby stocks are also used in cases of uncertainty about the offer of a product. The case of standby stocks is the speculative reserve, which comes from buying larger amounts of forecast data for shortages or revaluations of these items.

- **Dead Stock.** Dead stock or uninvolved stocks are products for which there is no demand at any particular time because either their lifetime or their life cycle (outdated in the market, obsolete) has expired. Businesses are required to monitor them carefully and to take action in time to remove them because they
bind space and make it difficult for the warehouse to operate. These include moving to other businesses, such as out-of-date feeds for reprocessing in feedstocks or even products ending their lifetime shortly in the future (which they call "short-lived" products) and which are also promoted for other uses, but also for social purposes.

- **Decoupling Stock.** Decoupling Stock or intermediate stock refers to the accumulated inventory that is between two or more inter-dependent operations, having the role of a buffer agent that is there to prevent potential breakdowns or unevenness in the production rates which gravely helps to reduce the need for output synchronization.

- **Seasonal Stock.** Seasonal stocks are created to absorb seasonal changes in raw material supply or demand. The level of the seasonal inventory at each point in time is given as the difference between cumulative production and total sales. More generally, there is an interaction between the cost of installing increased capacity that will under-perform low demand periods and the cost of maintaining seasonal stocks for long periods of time. Canned fruits or vegetables are an example of a seasonal stock created to cope with the seasonal availability of raw materials. Where seasonality can be predicted with satisfactory precision, linear programming is often used to calculate optimal capacity, optimum production schedule and optimum seasonal stock.

### 2.3 Costs associated with inventory

An important decision to be taken in relation to stocks is the level or amount of stocks and the accompanying costs of that stock. Every organization, in order to be sustainable, is obliged to minimize its costs but always in respect to the overall operational quality (Tersine & Tersine, 1988). The related costs generally are as follows:
Holding cost. This category includes the cost of capital stock, the cost of stock insurance, the cost of damage and losses, depreciation of the inventory and the cost of its handling during storage and transportation. In particular, the cost of frozen capital stems from the need for the firm to invest its capital to acquire the stocks and is equal to or greater than the return that the firm would have had if it had invested its capital in very low risk financial products. In many practical applications, the annual cost of maintaining the stock ranges from 20% to 35% of the value of the average stock held annually.

Order cost. It includes both fixed costs for placement of an order to the suppliers of the company (cost of reproductions) and the cost of buying the goods. It generally includes any costs that is accumulated in the procedure of a certain good appears to the warehouse of the organization. That means that costs like, printing paper, phone calls, man hours for handling the goods and storing them etc.

Shortage cost. If the stocks of a product are exhausted, the company is obliged to delay or cancel the order, thereby losing sales and negatively affecting the company's reputation. Losing sales is equal to losing money immediately whereas in the situation where the shortage of inventory resulted in the breakage of the relations with specific customers or even the damage of company's image, then the resulting loss of income is indirect, thus it cannot always be calculated accurately but it will be present in the forthcoming sales data.

Generally, as the amount of inventory increases, decreasing order frequency results in the following benefits (Radasanu, 2016):

- Increased availability, which means reducing deficits and improving customer satisfaction.
- Probable cost savings of the stock due to supplier discounts.
• Possible savings from a rise in purchase prices in the future.
• Reduce recursive order costs due to reduced number of orders.

2.4 ABC Analysis

A stock management system determines when an order should be made for a product and what quantity should be ordered. In most cases, the company manages a large number of different codes (SKU’s - products), for which they have to do different scheduling on their inventory. The classification methodology in the 'ABC' categories facilitates this process while still being widely applied in various areas.

In particular, this methodology separates the stock based on the fact that a relatively small number of products (about 20% of the total number of products) represents the bulk of the total value-reserved stock (about 75%-80%), also known as the Pareto law or rule 20-80 (Ravinder & Misra, 2014). In fact, the amount of the stock corresponds to the level of demand, i.e. the products that are demanded, are obviously purchased in larger quantities from the suppliers compared to the others demanded less, and consequently they are held in larger stocks. These products belong to the 'A' class

![Figure 1. Depiction of ABC sorting of products regarding total inventory value. Source: Collignon, J. & Vermorel, J. (2012)](image-url)
and are characterized as 'fast moving' products. In category 'B' belong the medium speeds, which have less demand and comprise about 20-25% of the total SKU’s, while the total value of their stock is approximately 15% of the total value of the stocks. Finally, the 'C' category includes slow-moving products (which are very low in demand), although they account for the bulk of the goods (about 60%), but the total value of their inventory represents around 5% -10% of the total value of the stocks (Tanwari, Lakhiar, & Shaikh, 2000). It should be noted here that these are approximations based on empirical data and sometimes some slow-moving products can also be classed as A class products as they might turn in considerable amounts of profit (Collignon & Vermorel, 2012).

The main objective of the 'ABC' classification is to determine the degree of control and monitoring of stocks. For example, for inventory of "A" products, their management system should be based on the strictest, control and continuous monitoring compared to the other two categories, in order to always keep sufficient inventory for the needs of customers. This has to be done because the fast-moving products are the most popular ones and the issue of their uninterrupted availability is therefore considered crucial (Viswanathan & Bhatnagar, 2005).

In addition, the company focuses on suppliers of Class A products, seeking to find the best value for money, because potential savings on the market for these products purchased in larger quantities are very important. The company must place particular emphasis on the purchase price negotiations with the suppliers of these products, as for example, a small discount per unit of product. Apart from cost, however, it is required, especially for products in this category, to meet stringent product quality standards and services (speed and accuracy) from trusted suppliers so that there are no delays and errors that will cost them at their disposal (Tanwari, Lakhiar, & Shaikh, 2000).
The flexibility of the company to find alternative suppliers immediately in emergencies is still a necessary critical parameter, in particular for suppliers of category A products. Therefore, a "supplier record" should be maintained to ensure a continuous smooth flow of products to the business (Tanwari, Lakhiar, & Shaikh, 2000).

Another ABC analysis application that can influence the warehouse operations is the product storage and order execution. Category A products, which are fast moving, are purchased and sold in large quantities. This implies that they cause frequent shifts in the warehouse by warehouse staff either to be settled in specific locations upon receipt or to be collected in the process of executing customer orders. In order to minimize staff distances and resulting greater productivity of warehouse processes (e.g., more orders made per hour), products in category 'A' are usually placed close to the entrance / exit of the warehouse (Millstein, Yang, & Li, 2014).

### 2.5 Forecasting Demand

A forecast is the estimate of future events to be used by an enterprise to program its plans. The ever-changing business conditions as a result of global competition and rapid technological change put pressure on businesses for as accurate a forecast as possible. Forecasts are needed to determine the company's resources, to plan the existing sources, and to get the rest that will be needed for production. Accurate forecasts allow businesses to make effective use of machinery, reduce production times and prepare inventories to fulfill the upcoming demand (Gardner, 1990). Therefore, through time there have been several forecasting methods that have been proposed.

Forecasting methods are categorized into two main categories, qualitative and quantitative. Qualitative forecasting methods include methods where one or more experts are making predictions based on their knowledge, experience and instinct. This
kind of prediction is subjective and includes the element of bias. On the other hand, quantitative methods are based on mathematical modeling and are therefore objective and repeatable (i.e., they produce the same result every time we import the same data). Quantitative methods require a number of numerical data that are not always available or reliable.

2.5.1 **Forecasting Methods**

Quantitative methods can be distinguished in those based on time series models and those based on causal models. The former presupposes that the necessary information for the forecast is contained in the data time series. Time series is a series of observations taken at regular intervals within a specified time period. The time series analysis assumes that predictions can be made based on the patterns of available data. Thus, this analysis looks for trends, seasonality, etc. in the data in order to create a prediction model (Chopra & Meindl, 2004).

On the other hand, causal models use a fairly different approach to predicting: they consider that the variable for which we want to predict is dependent in some way on one or more parameters. The difficulty lies in finding the mathematical relationship with which the requested variable is affected by these parameters (Chopra & Meindl, 2004).

It is obvious that causal models can be very complicated, especially when many parameters are taken into account. Qualitative are considered subjective and based on estimates and opinions. Quantitative are distinguished in methods of projection (time series analysis), causal methods and simulation. Time series analysis is based on the idea that historical observations can be used to predict future observations and deals with the
analysis of historical data on factors such as trend, seasonality, etc. mentioned above. In causal forecasting methods, it is assumed that demand depends on some external (macroeconomic) factors.

2.5.2 Quantitative Forecasting Methods

Quantitative forecasting methods are applied when the available information is quantitated in the form of numerical data and assuming that this behavioral pattern of historical data is retained-is followed in the future. Quantitative forecasting methods can be categorized according to the model used (Reid & Sanders, 2002).

2.5.2.1 Single Moving Average

This is the most basic prediction method used when demand is not fluctuating between observations and no seasonality element appears. The single moving average is the average of actual demand in recent months, as shown in formula (1):

\[ F_t = \frac{(D_{t-1} + D_{t-2} + \cdots + D_{t-N})}{N} \]  (1)

Where,

- \( F_t \) denotes the predicted demand for time \( t \)
- \( n \) = number of observations taken into account
- \( D_{t-i} \) = the demand observed at time \( t-i \) for \( i = 1, 2, \ldots, n \).
For every new known demand value, the forecasting formula (1) changes to (2):

\[ F_{t+1} = F_t + \frac{(D_t - D_{t-N})}{N} \]  

To determine the value of n for forecasting in a time series, the single moving average method is applied to the time series for different values of n and the value of that minimizing the predictive error is selected.

Also, it is worth noting that the term moving is used because every time a new observation becomes available in the time series, it replaces the oldest observation in the equation and a new moving average is calculated (Anderson, Sweeney, Williams, Camm, & Martin, 2011).

The single moving average is very useful to remove the random variability in the forecast when demand is not trending or seasonal. It is very important to select the appropriate interval length. The longer the selected interval, the more random the prediction and the impact of any extreme values. But when there is a trend in demand, which increases or decreases over time, the moving average of a long interval will smooth up the trend. Therefore, a shorter period of time, although more varied, follows more closely the demand trend.

2.5.2.2 Single exponential smoothing

Single exponential smoothing is one of the most widely known forecasting techniques. It is based on the fact that the most recent data is more important, whereas the older data (previous periods) are less important. It applies mainly to short-term planning and generally in cases where the forecast horizon is relatively small, and there
is no information available on any deterministic relationship linking the variable to be predicted and the independent factors that affect it. Its popularity is based on the fact that it is easy to apply, while the computing time requirements and data storage for its implementation are rather small (Reid and Sanders, 2002).

The forecast for the period t+1 is given by formula (3):

\[ F_{t+1} = a * D_t + (1 - a) * F_t \]  

(1)

Where,

- \( F_t \) denotes the predicted demand for time t
- \( a \) = is a smoothing variable between 0 and 1
- \( D_t \) = the demand observed at time t.

In order to be able to use this formula, first the forecast for period t has to be calculated.

The value of \( a \) is determined by both the experience of whoever makes the forecast and the qualitative characteristics of the predicted size. Considering that the data is relatively stable over time, then \( a \) will be given a small value (0.05-0.2). If, on the contrary, there are intense fluctuations in time, then \( a \) will get higher values.

### 2.5.2.3 Decomposition of a Time Series

A time series is defined as a dataset in chronological order that may include one or more elements of demand such as trend, seasonality, circularity, autocorrelation and randomness. The term "decomposition of a time series" means the division of a time series into its individual elements. In practice, it is relatively simple to find the trend and seasonality of a time series, but it is more difficult to identify circles, autocorrelation and random data. When demand also includes elements of trend and seasonality, the question is how these elements are related to each other. Seasonal fluctuation assumes
that the amount of seasonality is fixed irrespective of trend or average amount (Waters, 2003).

There are several components that can be observed in time series, some of which is the trend (T), cyclical component, seasonal component (S), irregular component and even sometimes calendar effects such as the Easter effect (Shumway & Stoffer, 2000).

*Figure 2. Plot of Original, Seasonal Adjusted and Trend series. Source: Shumway, R. & Stoffer D. (2000)*
2.5.2.4 Trend-based methods

When the demand is not constant then it must have a reason for that. One of the reasons is usually trends that appear throughout the data set. In that case, the formula that calculates the demand is the formula (4):

\[ D_t = S + T_t + e_t \]  

(2)

Figure 3. Components of a time series. Source: Shumway, R. & Stoffer D. (2000)
The corresponding time series that showcases a trend component can be observed in Figure 3 shown previously.

In order to forecast a data set that its time series showcases a linear trend, there are two methods, the Double moving average is the first one and the second one available is the Double exponential smoothing or else the Holt’s method (Waters, 2003).

### 2.5.2.4.1 Double Exponential Smoothing

The Holt’s method can be used when linear trend is present in time series and basically it the forecast is the summation of the series and trend as shown on formula (5):

\[ F_{t+1} = S_{t+1} + T_{t+1} \]  

(3)

Where,

- \( F_{t+1} \) = the forecast of time period \( t+1 \)
- \( S \) is the series of given time period
- \( T \) is the trend of given time period

The \( S \) and \( T \) in Holt’s method can be calculated by the formulas (6) and (7) respectively:

\[ S_1 = \frac{6}{n \times (n + 1)} \sum_{t=0}^{-n+1} t \times D_t + \frac{2 \times (2 \times n - 1)}{n \times (n + 1)} \sum_{t=0}^{-n+1} D_t \]  

(4)

\[ T_1 = \frac{12}{n \times (n^2 - 1)} \sum_{t=0}^{-n+1} t \times D_t + \frac{6}{n \times (n + 1)} \sum_{t=0}^{-n+1} D_t \]  

(5)

Where,

- \( S \) is the series of given time period
T is the trend of given time period

D is the demand of given time period

n is the number of data available

When the above formulas are exploited once, then for every new demand observation the new S and T values are calculated from formulas (8) and (9) respectively:

\[ S_{t+1} = a \times D_t + (1 - a) \times F_t \]  \hspace{1cm} (6)

\[ T_1 = b \times (S_{t+1} - S_t) + (1 - b) \times T_t \]  \hspace{1cm} (7)

Where,

\( a \) and \( b \) are smoothing constant values that can be calculated either empirically or from the minimization of the error functions that will be discussed later on. Generally, from the literature review, it can be observed that \( a \) ranges between 0.02 and 0.5 whereas \( b \) ranges between 0.005 and 0.17.

2.5.2.5 Seasonality Methods

As stated previously, time series can be exposed to seasonality and in that case, they have to be treated accordingly. A time series that is subject to seasonality is shown in Figure 3 above. Also, it is not necessary that a time series have both a trend and a seasonality component in it (Anderson, Sweeney, Williams, Camm, & Martin, 2011).
The mathematical formula of a time series that is subject to seasonality is formula (10):

\[ D_t = (S + t \cdot T) \cdot I_t + e_t \]  

(8)

Where,

- \( D_t \) is the demand data of a given time
- \( S \) is the series parameter
- \( T \) is the trend of the series
- \( e_t \) is the forecast error
- \( I_t \) is the seasonal index that can be calculated as follows:

\[ I = \frac{\text{Average value of season}}{\text{Grand average of all seasons}} \]  

(9)

2.5.2.5.1 Triple Exponential Smoothing

For the operations of triple exponential smoothing, Winter’s method is used to calculate the forecasted values. It follows the same procedures as Holt’s method but this time the index of seasonality is added which multiplies the sub of \( S \) and \( T \). The Winter’s method formula is (12):

\[ F_{t+1} = (S_{t+1} + T_{t+1}) \cdot I_{t+1} \]  

(10)

Same steps follow as with double exponential smoothing:
Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse

\[ S_1 = \frac{6}{n \times (n + 1)} \sum_{t=0}^{-n+1} t \times D_t + \frac{2 \times (2 \times n - 1)}{n \times (n + 1)} \sum_{t=0}^{-n+1} D_t \]  \hspace{1cm} (6)

\[ T_1 = \frac{12}{n \times (n^2 - 1)} \sum_{t=0}^{-n+1} t \times D_t + \frac{6}{n \times (n + 1)} \sum_{t=0}^{-n+1} D_t \]  \hspace{1cm} (7)

Where,

- \( S \) is the series of given time period
- \( T \) is the trend of given time period
- \( D \) is the demand of given time period
- \( n \) is the number of data available

When the above formulas are exploited once, then for every new demand observation the new \( S \) and \( T \) values are calculated from formulas (13) and (14) respectively as with Holt’s method, but this time the seasonality Index is added to the \( S \) function and also the following seasonality indices can me calculated:

\[ S_{t+1} = [a \times D_t + (1 - a) \times F_t] \times I_t \]  \hspace{1cm} (11)

\[ T_{t+1} = b \times (S_{t+1} - S_t) + (1 - b) \times T_t \]  \hspace{1cm} (12)

\[ I_{t+L} = c \times \frac{D_t}{S_t} + (1 - c) \times I_t \]  \hspace{1cm} (14)
\[ \sum_{t=1}^{L} I_t = L \]  \hspace{1cm} (13)

Where,

\( a \) and \( b \) are smoothing constant values that can be calculated either empirically or from the minimization of the error functions that will be discussed later on. Generally, from the literature review, it can be observed that \( a \) ranges between 0.02 and 0.5 whereas \( b \) ranges between 0.005 and 0.17.

\( C \) is another constant that varies between 0.05 and 0.5

\( L \) indicates the number of seasons that are being calculated

### 2.5.2.6 Forecast errors

Irrespective of the complexity of forecasting techniques, predicted demand will always be greater or lesser than the actual requirement and almost never equal to it. The difference between forecasting and actual demand is called a forecast error (Chopra & Meindl, 2004). The goal of the prediction is to minimize the error, therefore, if the error size is large this may mean either the prediction technique is wrong, or it needs modification in the parameters.

A cumulative prediction error measurement is required to test the performance of a prediction model and lead to a reassessment of all the standard parameters as required. Forecasts often involve errors. Forecast errors are statistically and randomly identified. Accidental errors are due to unpredictable factors affecting demand. On the
contrary, the statistical errors are related to the forecasting model and are due to poor estimation or omission of factors influencing demand, for example, seasonality (Anderson, Sweeney, Williams, Camm, & Martin, 2011).

The forecast error can be measured by comparing forecasts with actual demand prices. To forecast the demand for period \( t \) and \( D_t \) the real demand of period \( t \). The prediction error \( e_t \) is defined by the relationship:

\[
e_t = Y_t - F_t
\]  

(15)

Although since the data sets are more and more complex, some other, more specialized methods for forecast error estimations are present, with the most important of them being the Mean Absolute Deviation, the Mean Squared Error and the Mean Absolute Percentage Error (Kolassa & Siemsen, 2015):

**MAD – Mean Absolute Deviation:**

\[
MAD = \frac{\sum_{t=1}^{n} |D_t - F_t|}{n} = \sum_{t=1}^{n} \frac{|e_t|}{n}
\]

(16)

**MSE – Mean Squared Error:**

\[
MSE = \frac{\sum_{t=1}^{n} (D_t - F_t)^2}{n} = \sum_{t=1}^{n} \frac{e_t^2}{n}
\]

(17)

**MAPE – Mean Absolute Percentage Error:**

\[
MAPE = \frac{1}{n} * \sum_{t=1}^{n} \frac{|D_t - F_t|}{D_t} = \frac{1}{n} * \sum_{t=1}^{n} \frac{|e_t|}{D_t}
\]

(18)
2.6 Optimal Order Quantity

There are multiple approaches to estimate the optimal order quantity and the time period that each order should be placed in order to maximize its potential of saving costs. The simplest inventory models assume that the demand rate is deterministic and level, however that is rarely the case. Therefore, the problem of time-varying demand arises, to which there are three approaches that are used widely to tackle that problem. The Economic Order Quantity, the Wagner-Whitin Algorithm and several heuristic methods that offer close-to-optimal solutions (Silver, Pyke, & Thomas, 2016).

The best way to find the optimal solution to this problem is the use of Wagner-Whitin Algorithm, but it is not always the most efficient way as it is very complex and more often than not, a simple heuristic method or even the EOQ can offer solutions that vary only the slightest from the optimal solution of the algorithm.

2.6.1 Economic Order Quantity (EOQ)

Even though the Economic Order Quantity does not take into account time-varying demand, it can still be used for the determination of the optimal order quantity, simple by completely ignoring the time-variability. The formula of EOQ is as follows:

\[
EOQ = \sqrt{\frac{2 \times C_p \times R}{C_h}} \tag{19}
\]

Where,

\( C_p \) is the fixed order cost

\( C_h \) is the annual holding cost
$R$ is the annual demand

After the estimation of the optimal order cost, the cycle time can be found from the following formula:

$$T = \frac{Q}{R} \quad (20)$$

2.6.2 Heuristic Methods

The EOQ might be easy to use but it still does not take into account the varying element a demand usually showcases and there might be some cases that this would lead into troubles for the policy makers when it comes to the determination of order cycles and replenishment of the stock. The two best performing Heuristics as proposed by Baker (1989) are the Silver-Meal and Part-Period Balancing (PPB) (Baker, 1989).

2.6.3 Silver-Meal Method

This method, developed by Silver and Meal in 1973, is a simple variation of the basic EOQ but with a sensitivity to time-varying demand patterns. It basically considers a constant cost that is the fixed order cost $C_p$ and the total carrying cost to the end of period $T$ which is the holding cost (expressed in terms equal to a period) multiplied by the requirements that need to be met for the ongoing periods. Therefore, its purpose is to find the period that would generate the least amount of costs. As a mathematical formula it can be written as follows:

$$TRCUT(k) = \frac{C_p + \sum_{k=1}^{n} (k - 1) D(k) * C_h}{n} \quad (21)$$
Or, broken down step by step it goes as follows (Silver E., 2008):

Let the total relevant costs associated with a replenishment that lasts for T periods be denoted by TRC(k). These costs are composed of the fixed replenishment cost A and the inventory carrying costs. We wish to select T to minimize the total relevant costs per unit time, TRCUT(k), where

\[
TRCUT(k) = \frac{TRC(k)}{k} = \frac{A + \text{carrying costs}}{k}
\]

If k = 1, there are no carrying costs (we only replenish enough to cover the requirements of period 1), that is,

\[
TRCUT(1) = \frac{C_p}{1} = C_p
\]

If the setup cost \(C_p\) is large, this may be unattractive when compared with including the second period’s requirements in the replenishment, that is, using k = 2.

With k = 2, the carrying costs are \(D(2)C_h\), which are the cost of carrying the requirements \(D(2)\) for one period. Therefore,

\[
TRCUT(2) = \frac{C_p + D(2)C_h}{2}
\]

Now, the setup cost is apportioned across two periods, but a carrying cost is incurred. With k = 3 we still carry \(D(2)\) for one period, but now we also carry \(D(3)\) for two periods. Thus,

\[
TRCUT(3) = \frac{C_p + D(2)C_h + 2D(3)C_h}{3}
\]

In this case, the setup charge is apportioned across three periods.
The algorithm is being terminated when:

\[ TRCUT(k + 1) > TRCUT(k) \]
Therefore, at this point the associated k is the optimal number of periods that the first replenishment should cover in terms of stock. Then the algorithm is ran again to find the second and the rest ordering points (Silver, Pyke, & Thomas, 2016).

2.6.4 Part-Period Balancing (PPB)

The Part-Period Balancing heuristic method simply calculates the carrying costs, as they are calculated in the Silver-Meal method and terminates when the period that its carrying cost is closest to the fixed order cost is found. To elaborate further on this, when the absolute value of the difference between the carrying costs and the fixed order cost is at its minimum point, the algorithm terminates and picks the corresponding T as the optimal number for the first order point. Afterwards, the system is rerun for the second point and onwards.

The carrying cost can be calculated by the following formula:

\[
CarryingCost(k) = \sum_{k=1}^{n} (k - 1)D(k) \times C_h
\]

So, for every first period,

\[
CarryingCost(1) = 0
\]

Afterwards, for k=2:

\[
CarryingCost(1) = D(2) \times C_h
\]

For k=3:

\[
CarryingCost(3) = D(2) \times C_h + 2D(3) \times C_h
\]

And so on for each following period.
The algorithm breaks at a period when:

\[
|CarryingCost(k - 1) - C_p| > |CarryingCost(k) - C_p| \\
< |CarryingCost(k + 1) - C_p|
\]

### 2.7 Uncertainty in Inventory Management Systems

Simple inventory management models were based on the assumption that inventory demand and the timing of an order are known and stable. However, both cases are rarely met, as both demand and the execution time of orders are fluctuating. For this purpose, the firm should maintain a security stock to address unplanned demand. A reserve stock is defined as the amount of inventory maintained in addition to the expected demand (Jaksic, 2016). The existence of security stocks implies both a benefit and a cost to the business. Storage costs are due to the freezing of funds, the storage cost and the risk of damage or depreciation of the items. The benefit of security stocks is to eliminate the risk of lack of inventory in case of unexpected demand or delay in executing orders. As a result, the amount of the safety stock is determined by taking into account the benefit and the cost of storage (Williams, 1984).

#### 2.7.1 Fixed Order Quantity with Uncertain Demand – Continuous Review System

If demand is uncertain and inventory orders are repeated, the inventory management system should be adjusted to take account of the risk of non-satisfaction of demand due to lack of inventory. Two are the key factors to be determined, the fixed
amount of inventory to be ordered each time (Q) and the inventory level (R) where the order is made. Due to fluctuations in demand it is likely that either the demand during the lag time exceeds the stock quantity and the stock of the company is in short supply or the demand is less than the stock and therefore there is a surplus stock. Both cases involve costs for the business. The goal is to find optimal Q and R values that will minimize the expected total inventory management costs (Engelmeyer, 2016).

The hollow points of the peaks on the Figure 4 above indicate the lead time of each order from the moment it is placed to the moment that it arrives at the warehouses.

*Figure 5. Continuous Review System. Source: Engelmeyer, T. (2016)*
2.7.2 **Fixed Order Period System with Uncertain Demand – Periodic Review System**

In this system, new orders are triggered regardless of the amount of inventory at the end of a specified period. Thus, the ordering period is fixed but the order quantity is not fixed. Unlike the previous system we analyzed, inventory is counted only during re-run periods. Thus, these systems maintain larger inventory sizes to avoid a shortage between retention periods (Figure. 5)

![Fixed time between orders, fixed order quantity. Engelmeyer, T. (2016) (Figure 6)](image-url)
3 Drakoglou Auto-Parts Case

The examined company for the implementation of the Quantitative analysis of an Inventory Management System and the overall analysis of its aspects is a wholesale auto parts and lubricants seller, established in 1981 by Christos G. Drakoglou in Serres. The primary goal of the company is to provide the best of services to its clients and also offer high quality aftersales products at competing prices with great delivery experience.

It was in 1990 when Drakoglou decided to make the extra step and implemented an innovative – for the time – computerized system in order to standardize the services and increase the overall organization’s operations significantly, Due to its success and the need to service more customers and to have a higher capacity for more SKU’s, Drakoglou created a second store along with a brand new warehouse, at the core of the City of Thessaloniki, close to the rail station; combining the ease of access to the city center with plenty of space for the daily tasks of truck loading-unloading. With the addition of the Thessaloniki branch (which eventually became the main facility) the company expanded its reachability to central and southern Greece, even to Crete.

Drakoglou not only managed to withstand the rough situations, even during the economic crisis which, inevitably, forced many of the competitors to exit the market, but
also broadened its services in order to attract more clients and steadily build a larger clientele to unprecedented numbers for the size of the company. Even more, from 2013 and onwards, the company is constantly adding more SKU’s to its portfolio and also boasts exclusive product lines for the market of Greece. In sync with the steady growth of the company, an innovative B2B program has also been installed, thus standardizing and automating many procedures, in order to respond to its customer’s needs and allow them to order through that system with a more tangible interface than that of a phone order, and also to alleviate the workload of the phone center. Digitizing data also helps for the business intelligence part of the company, allowing robust analysis of data and gathering of vital information about the sustainability and overall workflow of the company.

More specifically, Drakoglou is specialized in a wide range of auto-parts for old and also new automobiles, with the most recent inclusion being the parts for hybrid or electrical cars which is a relatively new sector in Greece and is only represented by a handful of competitors. More importantly there are only a few of auto-shops that have the necessary skills and expertise to be able to maintain or fix a car of this technology.

The company’s mission statement is as follows (Drakoglou, 2017):

“We share the professional driver’s agony to make a living from his/her vehicle, the young boy’s/girl’s enthusiasm when sitting behind the wheel for the first time, the speed lover’s excitement, the family man’s concern for his children’s traveling safety, the everyday driver’s love for his/her car.”

3.1 Procedures within the organization

Placing order is being done roughly every week to ten days for most of the products and every 3 months for lubricants which come in large quantities. The ordering
process is being helped by the forecasting done using special algorithms and the data derived from the ERP program by the IT department. Most of the manufacturers that Drakoglou runs business with are from Greece but there are also many supplies and manufacturers from Germany, Italy, Spain, Belgium, Poland, Turkey and rarely from Taiwan and Japan on special occasions. Ordering from Taiwan or Japan can be tricky as they demand to be paid up front and the shipping time might take as much as two months, so a near-perfect forecast must be done beforehand. But, people are keener on buying something labeled as “Japanese” due to its quality, so most of the time if there is a tempting promotion from Japanese manufacturers it is worth the extra hassle.

Afterwards, when these orders are delivered from the suppliers, they must be sorted in the warehouse, which has to be done in a way that does not interfere with ongoing picking processes, so time management is of essence. In case a new line of products is order, it has to be imported in the ERP system by the allocated personnel with every detail possible and then find the appropriate location in the warehouse to store it, following the warehouse mapping regulations defined by the IT department for the optimal use of space. Regarding when there is shortage of a product that a customer asks, the procedure is to either reassure him that it will be delivered to him as soon as possible and offer him a 20% discount on the product or decline politely to alter the order cycle and lose that sale and potentially a part of the company’s “image”.

### 3.2 ABC Analysis

First and foremost, the most important task that has to be carried out is the ABC Analysis of the stock of Drakoglou Auto-Parts in order to determine the classes of the SKU’s which add up to 37682 product codes that have been active during the past 6 years (2013-2018). In the following Table, a portion of the class A products is presented.
As it can be seen on the table, the year 2017 was the one that was taken under consideration for the analysis of the products and the SKU’s that are 4th and 6th on the list and highlighted will be the ones used for the dissertation and the forthcoming analysis. The reasoning behind picking these two SKU’s is that first of all they are both Class A products, but also, they are basically the same product. 022-99901089 is the SKU code
that is being replaced by 022-30977089 in the company’s catalogues. This is a procedure that happened in 2017, therefore the former SKU code is no longer in the company’s catalogue. The Table 1 consists of the Revenue, Share % of the grand total of the company’s revenue for 2017, next is the column displaying the Accumulated Share % and lastly the column indicating the class of the products. In this case its apparent that every product is classified as class A since out of the 37626 only a few can be displayed, but it is worth noting that the pareto law of 80-20 holds for this data set with 6146 (~16% of products) belonging in the A class. The overall turnover of the company for the year of 2017 amounts close to 2 million €. Although it is worth noting that there is a plethora of C class SKU’s that are no longer in circulation and have 0 demand for the past couple of years and need to be withdrawn from the company’s records and be kept only as historical data, as it may disrupt future analyses when these codes pile up. These codes take up a significant portion of the total SKU pool with 18193 codes having 0 sales for the year 2017.

<table>
<thead>
<tr>
<th>Class</th>
<th>#</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6146</td>
<td>16%</td>
</tr>
<tr>
<td>B</td>
<td>6196</td>
<td>16%</td>
</tr>
<tr>
<td>C</td>
<td>25284</td>
<td>67%</td>
</tr>
<tr>
<td>Total</td>
<td>37626</td>
<td>100%</td>
</tr>
</tbody>
</table>

### 3.3 Forecasting the Demand

The SKU’s #022-99901089 & 022-30977089 refer to an antifreeze liquid produced by SWAG with a turnover of 6968.8€ for the former and 5886.3€ for the later for the year of 2017 which is only the 0.63% of the company’s total revenue for 2017. In total, data is available for the years 2013 through 2017 and the time series of which can be seen below.
of Figure 6. It should be noted here that from now own, both SKU codes will be treated as one.

![Figure 7 Time series for demand 2013-2017 for SKU# 022-99901089 + 022-30977089](image)

From the above display of time series, it is distinctive that there is seasonality and a trend throughout the time series. In the late months of each year and the beginning of the next there is a spike in demand which can be explained from the fact that this is the period when most people refill their antifreeze liquid after some months of winter, in addition to the extra money that they receive from the Christmas bonus salary.

The exact demand values can be seen below in table 3.

<table>
<thead>
<tr>
<th></th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>254</td>
<td>0</td>
<td>520</td>
<td>654</td>
<td>668</td>
</tr>
<tr>
<td>February</td>
<td>335</td>
<td>230</td>
<td>214</td>
<td>234</td>
<td>179</td>
</tr>
<tr>
<td>March</td>
<td>62</td>
<td>40</td>
<td>287</td>
<td>54</td>
<td>137</td>
</tr>
<tr>
<td>April</td>
<td>3</td>
<td>136</td>
<td>80</td>
<td>43</td>
<td>214</td>
</tr>
<tr>
<td>May</td>
<td>57</td>
<td>22</td>
<td>164</td>
<td>328</td>
<td>128</td>
</tr>
<tr>
<td>June</td>
<td>285</td>
<td>26</td>
<td>47</td>
<td>32</td>
<td>95</td>
</tr>
<tr>
<td>July</td>
<td>41</td>
<td>58</td>
<td>68</td>
<td>213</td>
<td>130</td>
</tr>
<tr>
<td>August</td>
<td>41</td>
<td>139</td>
<td>247</td>
<td>64</td>
<td>234</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>September</th>
<th>370</th>
<th>494</th>
<th>390</th>
<th>157</th>
<th>306</th>
</tr>
</thead>
<tbody>
<tr>
<td>October</td>
<td>436</td>
<td>398</td>
<td>366</td>
<td>407</td>
<td>868</td>
</tr>
<tr>
<td>November</td>
<td>574</td>
<td>425</td>
<td>348</td>
<td>842</td>
<td>1186</td>
</tr>
<tr>
<td>December</td>
<td>448</td>
<td>669</td>
<td>498</td>
<td>1026</td>
<td>1381</td>
</tr>
</tbody>
</table>

Table 3. Demand Values 2013-2017

It is now clearer that there is a trend component in this time series as most of the values for each month increase along with the years (which can be seen in the annual demand Table 4).

<table>
<thead>
<tr>
<th></th>
<th>022-99901089 &amp; 30977089</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>2906</td>
</tr>
<tr>
<td>2014</td>
<td>2637</td>
</tr>
<tr>
<td>2015</td>
<td>3229</td>
</tr>
<tr>
<td>2016</td>
<td>4054</td>
</tr>
<tr>
<td>2017</td>
<td>5526</td>
</tr>
</tbody>
</table>

Table 4 Annual Demand

In order to make sure that there is a trend component in the time series, a Mann-Kendall two-tailed trend test should be run. The Mann-Kendall Test is a non-parametric, distribution free test that is usually applied in order to investigate a series for monotonic trend and even seasonality with some variations. It compares every point of a time series with the points above that and below, therefore one of the restrictions is that there must be no coagulated points. Afterwards, depending on the points evaluated, the null hypothesis H0, which states that data are identically distributed, can be rejected in favor of hypothesis H1 which states that there is a monotonic trend.

The initial test for trend has been run with the following results:

Summary statistics:
Mann-Kendall trend test / Two-tailed test (022-99901089 & 30977089):

Kendall's tau 0.188

S = 333.000

Var(S) = 24580.333

p-value (Two-tailed) = 0.034

alpha = 0.05

The p-value is computed using an exact method.

Test interpretation:

H0: There is no trend in the series

Ha: There is a trend in the series

As the computed p-value is lower than the significance level alpha=0.05, one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.

The risk to reject the null hypothesis H0 while it is true is lower than 3.42%.

Therefore, there is indeed a trend component in the exploited time series.

Also, a Seasonal Kendall test has been ran, which is a special case of the Mann Kendall Trend Test showcased above, in order to reassure that there is also a seasonality component in the time series. The period was set to 12 as it is what can be observed in the timeseries. So, the test will investigate whether there is a “trend” of a 12 period season in our timeseries.

Seasonal Mann-Kendall Test / Period = 12 / Serial independence / Two-tailed test (022-99901089 & 30977089):
Kendall's tau 0.283

\[ S' = 34.000 \]

\[ \text{Var}(S') = 200.000 \]

p-value (Two-tailed) = 0.020

\[ \alpha = 0.05 \]

The p-value is computed using an exact method.

Test interpretation:

H0: There is no trend in the series

Ha: There is a trend in the series

As the computed p-value is lower than the significance level \( \alpha = 0.05 \), one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.

The risk to reject the null hypothesis H0 while it is true is lower than 1.96%.

The next step is to calculate the forecast of demand for the year 2018 using the known formula (12):

\[ F_{t+1} = (S_{t+1} + T_{t+1}) \times I_{t+1} \]

In order to calculate the forecast, the seasonal factors \( I_{t+1} \) need to be calculated for the total of years there is available data, which are years 2013 through 2017. Although,
in order to calculate the seasonal factors, the mean demand of every cycle has to be calculated.

In our case and due to the peculiarity of the data set, a different than normal approach was chosen. After close investigation of the values, specific seasons could be distinguished. Four seasons have been created, which consist of:

<table>
<thead>
<tr>
<th>Season</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>January &amp; December</td>
</tr>
<tr>
<td>I2</td>
<td>February &amp; March</td>
</tr>
<tr>
<td>I3</td>
<td>April - August</td>
</tr>
<tr>
<td>I4</td>
<td>September - November</td>
</tr>
</tbody>
</table>

The mean cycle is calculated by finding the average of the corresponding monthly demand, which can be witnessed on Table 4 above. Every value on the Mean Dt column calculates the average of its own Dt and the 5 values above that and the 6 values below that, which means that Mean Dt of June 2013 calculates the average of January 2013 through December 2013.

Although, due to the fact that the amount of data entries for each cycle is an even number and there no middle value, another variable must be calculated, that of the Centered Dt which is equal to the average value between the same row’s Mean Dt and that of its previous row. For example, the Centered Dt of July is the average value between Mean Dt of July and June. Lastly, on the far-right column the seasonal factor can be calculated by dividing the Dt of a month by the same month’s Centered Dt.

<table>
<thead>
<tr>
<th>Month</th>
<th>Dt</th>
<th>Mean Dt</th>
<th>Centered Dt</th>
<th>Seasonal Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-13</td>
<td>254</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Month</td>
<td>Quantity</td>
<td>Sales 1</td>
<td>Sales 2</td>
<td>Difference</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>---------</td>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>Feb-13</td>
<td>335</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mar-13</td>
<td>62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apr-13</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>May-13</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jun-13</td>
<td>285</td>
<td>242.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jul-13</td>
<td>41</td>
<td>221.00</td>
<td>231.58</td>
<td>0.17704</td>
</tr>
<tr>
<td>Aug-13</td>
<td>41</td>
<td>212.25</td>
<td>216.63</td>
<td>0.18927</td>
</tr>
<tr>
<td>Sep-13</td>
<td>370</td>
<td>210.42</td>
<td>211.33</td>
<td>1.75079</td>
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<td>436</td>
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<td>215.96</td>
<td>2.01891</td>
</tr>
<tr>
<td>Nov-13</td>
<td>574</td>
<td>218.58</td>
<td>220.04</td>
<td>2.60860</td>
</tr>
<tr>
<td>Dec-13</td>
<td>448</td>
<td>197.00</td>
<td>207.79</td>
<td>2.15601</td>
</tr>
<tr>
<td>Jan-14</td>
<td>0</td>
<td>198.42</td>
<td>197.71</td>
<td>0.00000</td>
</tr>
<tr>
<td>Feb-14</td>
<td>230</td>
<td>206.58</td>
<td>202.50</td>
<td>1.13580</td>
</tr>
<tr>
<td>Mar-14</td>
<td>40</td>
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<td>211.75</td>
<td>0.18890</td>
</tr>
<tr>
<td>Apr-14</td>
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<td>Jul-14</td>
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<td>241.42</td>
<td>0.24025</td>
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<tr>
<td>Aug-14</td>
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<td>261.75</td>
<td>262.42</td>
<td>0.52969</td>
</tr>
<tr>
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<td>494</td>
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<td>272.04</td>
<td>1.81590</td>
</tr>
<tr>
<td>Oct-14</td>
<td>398</td>
<td>277.67</td>
<td>280.00</td>
<td>1.42143</td>
</tr>
<tr>
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<td>283.58</td>
<td>1.49868</td>
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<td>291.67</td>
<td>1.78286</td>
</tr>
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<td>296.58</td>
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</tr>
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<td>0.96714</td>
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<td>Jul-15</td>
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<td>280.25</td>
<td>274.67</td>
<td>0.24757</td>
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<td>Aug-15</td>
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<td>281.08</td>
<td>0.87874</td>
</tr>
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<td>390</td>
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<td>272.21</td>
<td>1.43273</td>
</tr>
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<td>Oct-15</td>
<td>366</td>
<td>259.42</td>
<td>260.96</td>
<td>1.40252</td>
</tr>
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</table>
Now that all the Seasonal Factors have been calculated, the next step is to evaluate the average seasonal factors for each month.

<table>
<thead>
<tr>
<th>Month</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov-15</td>
<td>348</td>
<td>273.08</td>
</tr>
<tr>
<td>Dec-15</td>
<td>498</td>
<td>271.83</td>
</tr>
<tr>
<td>Jan-16</td>
<td>654</td>
<td>283.92</td>
</tr>
<tr>
<td>Feb-16</td>
<td>234</td>
<td>268.67</td>
</tr>
<tr>
<td>Mar-16</td>
<td>54</td>
<td>249.25</td>
</tr>
<tr>
<td>Apr-16</td>
<td>43</td>
<td>252.67</td>
</tr>
<tr>
<td>May-16</td>
<td>328</td>
<td>293.83</td>
</tr>
<tr>
<td>Jun-16</td>
<td>32</td>
<td>337.83</td>
</tr>
<tr>
<td>Jul-16</td>
<td>213</td>
<td>339.00</td>
</tr>
<tr>
<td>Aug-16</td>
<td>64</td>
<td>344.42</td>
</tr>
<tr>
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<td>157</td>
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<tr>
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<td>407</td>
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</tr>
<tr>
<td>Nov-16</td>
<td>842</td>
<td>338.92</td>
</tr>
<tr>
<td>Dec-16</td>
<td>1026</td>
<td>344.17</td>
</tr>
<tr>
<td>Jan-17</td>
<td>668</td>
<td>337.25</td>
</tr>
<tr>
<td>Feb-17</td>
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<td>363.83</td>
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<td>Jun-17</td>
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<td>234</td>
</tr>
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<td>Aug-17</td>
<td>234</td>
<td>306</td>
</tr>
<tr>
<td>Sep-17</td>
<td>868</td>
<td>646</td>
</tr>
<tr>
<td>Oct-17</td>
<td>1186</td>
<td>1381</td>
</tr>
</tbody>
</table>

Table 7 Seasonal Factors

<table>
<thead>
<tr>
<th>I1</th>
<th>1.9236</th>
</tr>
</thead>
</table>

Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse
The sum of the seasonal factors is 4.5302 even though according to formula (15) it should be equal to L, which is 4. Therefore, in order to normalize that factor, every seasonal factor has to be multiplied by the ratio of \( \frac{4}{4.5302} \) which will end up as follows:

<table>
<thead>
<tr>
<th>Seasonal Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>1.698467</td>
</tr>
<tr>
<td>I2</td>
<td>0.548737</td>
</tr>
<tr>
<td>I3</td>
<td>0.331666</td>
</tr>
<tr>
<td>I4</td>
<td>1.42113</td>
</tr>
</tbody>
</table>

**Table 8. Normalized Seasonal Factors**

Following the above calculation, now is the time to calculate the Trend by revisiting Table 4 and adding three more columns that are necessary for the calculation of trend.
## Quantitative Analysis of Inventory Management System - Case Study of an Auto-Parts Warehouse

<table>
<thead>
<tr>
<th>Month</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>May-13</td>
<td>57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jun-13</td>
<td>285</td>
<td>242.17</td>
<td>103.87</td>
</tr>
<tr>
<td>Jul-13</td>
<td>41</td>
<td>221.00</td>
<td>231.58</td>
</tr>
<tr>
<td>Aug-13</td>
<td>41</td>
<td>212.25</td>
<td>216.63</td>
</tr>
<tr>
<td>Sep-13</td>
<td>370</td>
<td>210.42</td>
<td>211.33</td>
</tr>
<tr>
<td>Oct-13</td>
<td>436</td>
<td>221.50</td>
<td>215.96</td>
</tr>
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<td>Nov-13</td>
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<td>220.04</td>
</tr>
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<td>197.00</td>
<td>207.79</td>
</tr>
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<td>197.71</td>
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<tr>
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<td>40</td>
<td>216.92</td>
<td>211.75</td>
</tr>
<tr>
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<td>136</td>
<td>213.75</td>
<td>215.33</td>
</tr>
<tr>
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<td>22</td>
<td>201.33</td>
<td>207.54</td>
</tr>
<tr>
<td>Jun-14</td>
<td>26</td>
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</tr>
<tr>
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<td>58</td>
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<tr>
<td>Aug-14</td>
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</tr>
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<tr>
<td>Nov-14</td>
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<td>283.58</td>
</tr>
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<td>Dec-14</td>
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<td>290.38</td>
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<td>Jan-15</td>
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</tr>
<tr>
<td>Mar-15</td>
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<td>296.75</td>
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<td>Jun-15</td>
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<td>234</td>
<td>268.67</td>
<td>276.29</td>
</tr>
</tbody>
</table>

PASCHALERIS GEORGIOS
Now that the three new parameters are known, the calculation of Trend and Seasonality can occur by using the formulas 6 and 7.

\[
S_1 = \frac{6}{n \times (n+1)} \sum_{t=0}^{n+1} t \times D_t + \frac{2 \times (2 \times n - 1)}{n \times (n+1)} \sum_{t=0}^{n+1} D_t
\]  

(6)

\[
T_1 = \frac{12}{n \times (n^2 - 1)} \sum_{t=0}^{n+1} t \times D_t + \frac{6}{n \times (n+1)} \sum_{t=0}^{n+1} D_t
\]  

(7)
Since now these two sums are known,

\[
\sum_{t=0}^{n+1} D_t = 0
\]
\[
\sum_{t=0}^{n+1} t \cdot D_t = 0
\]

S1 and T1 can be calculated:

\[S1 = 570.3\]
\[T1 = 4.3294\]

Now that S1 and T1 are known forecasting the demand is viable via the following formula:

\[F_t = (S + t \cdot T) \cdot I_t\] (22)

Where S and T are constants equal to S1 and T1 respectively. The varying parameters are \(F_t\) and \(I_t\).

For January through December 2018 the forecasted values are:

\[\begin{array}{c}
F_1 & 976.00 \\
F_2 & 317.70 \\
F_3 & 320.08 \\
F_4 & 194.89 \\
F_5 & 196.33 \\
F_6 & 197.77 \\
F_7 & 199.20 \\
F_8 & 200.64 \\
F_9 & 865.85 \\
F_{10} & 872.01 \\
F_{11} & 878.16 \\
F_{12} & 1056.89
\end{array}\]

Table 10. Forecasted Demand for 2018
Now that the forecasted demand of 2018 is available, the next step is to compare this to the actual demand of 2018 and calculate the error using the Mean Absolute Deviation (MAD) formula (18):

\[
MAD = \frac{1}{n} \sum_{t=1}^{n} |D_t - F_t| = \frac{1}{n} \sum_{t=1}^{n} |e_t| \quad (18)
\]

Furthermore, the calculation of MSE (20) is reasonable to be shown here, as it will also be used as an indicator of the standard deviation (\(\sigma\)).

\[
MSE = \frac{1}{n} \sum_{t=1}^{n} (D_t - F_t)^2 = \frac{1}{n} \sum_{t=1}^{n} e_t^2 = \sigma^2
\]

The Table 9 is used to display the initial findings:

| Year | Month | Forecast | Actual Demand | \(|D_t - F_t|\) | \((D_t-F_t)^2\) | MAD | MSE |
|------|-------|----------|---------------|----------------|----------------|-----|-----|
| 2018 | January | 976.00 | 401 | 575.00 | 330623.3 | 183.23 | 57002 |
|      | February | 317.70 | 117 | 200.70 | 40280.28 |       |     |
|      | March | 320.08 | 178 | 142.08 | 20185.37 |       |     |
|      | April | 194.89 | 301 | 106.11 | 11258.34 |       |     |
|      | May | 196.33 | 222 | 25.67 | 658.9163 |       |     |
|      | June | 197.77 | 169 | 28.77 | 827.5158 |       |     |
|      | July | 199.20 | 163 | 36.20 | 1310.622 |       |     |
|      | August | 200.64 | 510 | 309.36 | 95704.57 |       |     |
|      | September | 865.85 | 724 | 141.85 | 20122.21 |       |     |
|      | October | 872.01 | 723 | 149.01 | 22202.64 |       |     |
|      | November | 878.16 | 745 | 133.16 | 17731.12 |       |     |
|      | December | 1056.89 | 706 | 350.89 | 123121.4 |       |     |
|      | sum= | 2198.78 | 684026.2 |       |       |     |     |
On the graph (Figure 8) below the error is visualized by showing the time series of both the forecasted demand and the actual demand.

![Forecast vs Actual Demand](image)

*Figure 8. Visualization of the error between actual demand and forecast*

The error value of MAD needs to be further reduced and that can be achieved by using the Winter’s Method that introduces the smoothing constants of $\alpha$, $b$ and $c$ as explained in chapter 2. The MAD value seems to be on the high side at the moment. By using these smoothing constants, one is allowed to calculate the new values of $S$ and $T$ using the following formulas:

\[
S_{t+1} = [a \cdot D_t + (1 - a) \cdot F_t] \cdot I_t
\]

\[
T_{t+1} = b \cdot (S_{t+1} - S_t) + (1 - b) \cdot T_t
\]
\[ I_{t+L} = c \cdot \frac{D_t}{S_t} + (1 - c) \cdot I_t \]  

(16)

Before proceeding with the calculation, it is needed to set a value for each smoothing constant. After a reverse engineering process of trial and error in excel, the optimal smoothing constants that would greatly reduce the errors seemed to be the following:

\[ \alpha = 0.1637 \]
\[ b = 0.17 \]
\[ c = 0.5 \]

There were restrictions on the calculation of these variable that were solely based of the respective bibliography\(^1\).

S1, T1, and F1 are now known, so by exploiting formulas 15, 16 and 18 the rest of the data can be filled in, thus finding the new re-evaluated forecasted demand values. At this point it is considered that all data for 2018 is known, which allows to fill in the whole table. F is calculated by the revised formula 12 for any additional periods that the demand is not known:

\[ F_t = (S_t + t \cdot T_t) \cdot I_t \]  

(12b)

Table 12. Evaluated values of S, T, F and I

<table>
<thead>
<tr>
<th>t</th>
<th>S</th>
<th>T</th>
<th>F</th>
<th>I</th>
</tr>
</thead>
</table>

\(^1\) Jacobs & Chase (2013) and Schroeder, Rungtusanatham, & Goldstein (2013) recommend values for \( \alpha \) between 0.10 and 0.30. Stevenson (2012) and Heizer & Render (2011) support a range of 0.05 – 0.50. Chopra & Meindl (2013) limit the value of \( \alpha \) to 0.20.
A note should be added here, due to the immense amount of calculations, they cannot be shown on paper, therefore the complete table (table 12) is shown. Every time there was a new 2018 becoming known, F, T, S, I and normalized I were being recalculated.

In the following table the results of the re-evaluated forecast and the respective new error can be witnessed.

Table 13. Error evaluation of new re-evaluated forecast values

| Year | Month | Initial Forecast | Actual Demand | Forecast | |Dt-Ft| (Dt-Ft)^2 | MAD | MSE |
|------|-------|------------------|---------------|---------|---|-----------|-----|-----|
| 2018 | January | 976.00 | 401 | 740.19 | 339.19 | 115046.569 | 121.5796 | 27331.23 |
|      | February | 317.70 | 117 | 322.20 | 205.20 | 42105.3942 |        |     |
|      | March   | 320.08 | 178 | 200.17 | 22.17 | 491.632725 |        |     |
|      | April   | 194.89 | 301 | 169.85 | 131.15 | 17199.7462 |        |     |
|      | May     | 196.33 | 222 | 246.83 | 24.83 | 616.50082 |        |     |
As it can be observed, the new error value has been reduced indicating that the method being followed is effective.

Also, as it has been previously performed, the seasonal factors have to be normalized again as they have been re-evaluated.

Table 14. Normalization of re-evaluated seasonal factors

<table>
<thead>
<tr>
<th>Seasonal Factors</th>
<th>Normalized Seasonal Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>i13</td>
<td>1.3503</td>
</tr>
<tr>
<td>i14</td>
<td>0.4013</td>
</tr>
<tr>
<td>i15</td>
<td>0.7835</td>
</tr>
<tr>
<td>i16</td>
<td>1.4276</td>
</tr>
<tr>
<td>sum</td>
<td>3.9627</td>
</tr>
</tbody>
</table>

The finalized graph comparing the re-evaluated forecast to the initial forecast and their deviation from the actual demand follows.
3.4 Evaluation of Costs

As mentioned in chapter 2, behind a product there are various very important costs that must be addressed. Many times, they are not given the attention they deserve, resulting in big loss of capital for the company.

Therefore, it is a vital part of the inventory management system that is being constructed for the present dissertation, to address these costs and estimate them as closely as possible, thus making the inventory management system as accurate as possible.

The main cost components that will be evaluated are the following:

- $C_h$: holding cost
- $C_u$: shortage cost per unit
- $C_p$: fixed order cost
3.4.1 Holding cost

The holding cost is equal to the acquisition cost \((p)\) of the product multiplied by the capital cost \((I)\). In the case of Drakoglou, it is estimated a percentage of 18% that includes the capital cost. On top of that, the average acquisition cost for the SKU that is being investigated is 1.79\(\, \text{€ per unit}\).

Therefore, the holding cost equals:

\[
c_h = I \times p = 0.18 \times 1.79 = 0.32\, \text{€ per unit of antifreeze bottle annually.}
\]

3.4.2 Shortage Cost

The determination of shortage cost is oftentimes hard to manage. Other than some specific variables, most of the components that make up the shortage cost can be considered abstract. When an order is not fulfilled there is the obvious damage to the brand image that is hard to measure and in the case of Drakoglou there have been no research on that matter, therefore it will not be taken under consideration.

What Drakoglou – Auto Parts does when there is a stock out is to procure the products from the inner wholesale market and then resell it immediately, usually with close to no profit, but it can be considered a safety measure to avoid discrepancies with the clients. This can be translated to a loss of revenue, since it bears no profit. The profit that the company is looking for to make from each SKU in average is 20%, therefore the shortage cost can be evaluated as 20% of the sell price of the SKU that is being investigated which is 2.75\(\, \text{€} \). So, the shortage cost is:

\[
c_s = 2.75 \times 0.2 = 0.55\, \text{€ per unit short}
\]
3.4.3 **Fixed Order Cost**

The fixed order cost is comprised of the cost of workhours that are being spent on an order in total and the cost of transportation for the order. The latter is easily accessible since the firm has an agreement with the supplier that has a set value for each order at 20€. Also, for the completion of an order the workhours spent are divided as follows:

- 15 minutes for 1 person in IT to prepare and forward the order to the administration
- 1 hour for 1 person at the administration to double check the order, check the market for alternatives and reconsider the order.
- 45 minutes for 2 people at the warehouse to receive the order, check the accompanying documents, check the goods for any faulty products or wrong SKU’s, stock them at the warehouse, prepare the documents and proceed with transferring the documents to the IT department
- 15 minutes for 1 person in the IT department to digitize the documents and update the system.

In total, it is 3 workhours spent on each order. The average salary is 1100€, which equals to 6.25€ per hour. The cost of the manhours spent for every order is 18.75, which takes the total fixed cost up to:

\[ C_p = 38.75€. \]

### 3.5 Order Quantity and Cost Calculation

The firm is still using the periodic review system for the SKU’s that are subject to frequent high demand. Orders are being placed every 2 months for the SKU’s that is
under investigation with a lead time that is usually a whole month (L=30), whereas the level of the maximum inventory (S) is easily estimated by empirical factors.

For the model that is being formulated for this thesis, two parameters need to be addressed, the time interval of two successive orders (T) and the value of S, the maximum level of inventory.

### 3.5.1 Estimations With Seasonality

Since there is the seasonality component, it should be taken into consideration, mainly regarding the max inventory levels, which must fluctuate depending on the season, in order to cover the increased demand is periods following such pattern (Figure 10). For the following estimations, the following are being applied:

- The forecasted data for year 2018 is used for the calculations
- S is the maximum level of inventory for every cycle that is varying
- $\mu(L+T)$ is the average value of the forecasted demand for every time interval $L+T$
- $\sigma = \mu/\alpha$ for every time interval $L+T$
- where $\alpha$ is a constant number that is derived from dividing $\sigma$ over $\mu$ for the period 2018.
- Data is treated as it is following a normal distribution as this is generally accepted in the literature regarding safety stock calculation.
Before proceeding, the data of periods with known demand 2017-2018 is used to extract the mean value and also a constant number $\alpha$ that links together $\mu$ and $\sigma$ values, which will be used later to calculate the necessary data for forecasted period 2018.

<table>
<thead>
<tr>
<th>Month</th>
<th>Dt</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>740.19</td>
</tr>
<tr>
<td>February</td>
<td>322.20</td>
</tr>
<tr>
<td>March</td>
<td>200.17</td>
</tr>
<tr>
<td>April</td>
<td>169.85</td>
</tr>
<tr>
<td>May</td>
<td>246.83</td>
</tr>
<tr>
<td>June</td>
<td>226.87</td>
</tr>
<tr>
<td>July</td>
<td>186.64</td>
</tr>
<tr>
<td>August</td>
<td>165.42</td>
</tr>
<tr>
<td>September</td>
<td>858.51</td>
</tr>
<tr>
<td>October</td>
<td>812.34</td>
</tr>
<tr>
<td>November</td>
<td>782.98</td>
</tr>
<tr>
<td>December</td>
<td>754.51</td>
</tr>
</tbody>
</table>
\[ \mu = 456 \]
\[ \sigma = 165.32 = \sqrt{MSE} \]
\[ \alpha = \frac{\mu}{\sigma} = 2.755 \]

For the purpose of the following chapters, some more forecasted periods are going to be needed, but since the actual demand is not available, the forecasting of the additional periods (January 19 through June 19) was based on the last known data using the following formula:

\[ F_t = (S + t \times T) \times I_t \]

Were $S$ and $T$ have the last known values calculated during the forecasting procedure.

<table>
<thead>
<tr>
<th>Ft</th>
<th>Month</th>
<th>Dt</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Jan-18</td>
<td>740</td>
</tr>
<tr>
<td>F2</td>
<td>Feb-18</td>
<td>322</td>
</tr>
<tr>
<td>F3</td>
<td>Mar-18</td>
<td>200</td>
</tr>
<tr>
<td>F4</td>
<td>Apr-18</td>
<td>170</td>
</tr>
<tr>
<td>F5</td>
<td>May-18</td>
<td>247</td>
</tr>
<tr>
<td>F6</td>
<td>Jun-18</td>
<td>227</td>
</tr>
<tr>
<td>F7</td>
<td>Jul-18</td>
<td>187</td>
</tr>
<tr>
<td>F8</td>
<td>Aug-18</td>
<td>165</td>
</tr>
<tr>
<td>F9</td>
<td>Sep-18</td>
<td>859</td>
</tr>
<tr>
<td>F10</td>
<td>Oct-18</td>
<td>812</td>
</tr>
<tr>
<td>F11</td>
<td>Nov-18</td>
<td>783</td>
</tr>
<tr>
<td>F12</td>
<td>Dec-18</td>
<td>755</td>
</tr>
<tr>
<td>F13</td>
<td>Jan-19</td>
<td>740</td>
</tr>
<tr>
<td>F14</td>
<td>Feb-19</td>
<td>252</td>
</tr>
<tr>
<td>F15</td>
<td>Mar-19</td>
<td>254</td>
</tr>
</tbody>
</table>
Although, in an effort to gain a bit more flexibility and thus hopefully end up with more accurate results, the split of each month to two equal periods has been decided. Therefore, instead of 12 periods, we now have 24 (with the appropriate extensions). The new 15-day table is the following.

**Table 17. Table 16 but split in 15day periods**

<table>
<thead>
<tr>
<th>Forecast 2018</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-18</td>
<td>370</td>
<td></td>
</tr>
<tr>
<td>Jan-18</td>
<td>370</td>
<td></td>
</tr>
<tr>
<td>Feb-18</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>Feb-18</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>Mar-18</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Mar-18</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Apr-18</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>Apr-18</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>May-18</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>May-18</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>Jun-18</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>Jun-18</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>Jul-18</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>Jul-18</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>Aug-18</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>Aug-18</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>Sep-18</td>
<td>429</td>
<td></td>
</tr>
<tr>
<td>Sep-18</td>
<td>429</td>
<td></td>
</tr>
<tr>
<td>Oct-18</td>
<td>406</td>
<td></td>
</tr>
<tr>
<td>Oct-18</td>
<td>406</td>
<td></td>
</tr>
<tr>
<td>Nov-18</td>
<td>391</td>
<td></td>
</tr>
<tr>
<td>Nov-18</td>
<td>391</td>
<td></td>
</tr>
<tr>
<td>Dec-18</td>
<td>377</td>
<td></td>
</tr>
<tr>
<td>Dec-18</td>
<td>377</td>
<td></td>
</tr>
</tbody>
</table>

*Table 17. Table 16 but split in 15day periods*
Since value a is now known and we have the complete table that will be used for the rest of the dissertation, we need to calculate the standard deviation for each period, given the fact that each new Forecasted value that becomes known is $\mu(L+T)$.

<table>
<thead>
<tr>
<th>Forecast 2018</th>
<th>$\mu(L+T)$</th>
<th>$\sigma(L+T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-18</td>
<td>370</td>
<td>134.31</td>
</tr>
<tr>
<td>Jan-18</td>
<td>370</td>
<td>134.31</td>
</tr>
<tr>
<td>Feb-18</td>
<td>161</td>
<td>58.46</td>
</tr>
<tr>
<td>Feb-18</td>
<td>161</td>
<td>58.46</td>
</tr>
<tr>
<td>Mar-18</td>
<td>100</td>
<td>36.32</td>
</tr>
<tr>
<td>Mar-18</td>
<td>100</td>
<td>36.32</td>
</tr>
<tr>
<td>Apr-18</td>
<td>85</td>
<td>30.82</td>
</tr>
<tr>
<td>Apr-18</td>
<td>85</td>
<td>30.82</td>
</tr>
<tr>
<td>May-18</td>
<td>123</td>
<td>44.79</td>
</tr>
<tr>
<td>May-18</td>
<td>123</td>
<td>44.79</td>
</tr>
<tr>
<td>Jun-18</td>
<td>113</td>
<td>41.17</td>
</tr>
<tr>
<td>Jun-18</td>
<td>113</td>
<td>41.17</td>
</tr>
<tr>
<td>Jul-18</td>
<td>93</td>
<td>33.87</td>
</tr>
<tr>
<td>Jul-18</td>
<td>93</td>
<td>33.87</td>
</tr>
<tr>
<td>Aug-18</td>
<td>83</td>
<td>30.02</td>
</tr>
</tbody>
</table>
Also, it should be noted here that even though the lead time is an existing variable, the orders are placed in such a manner that the delivery date coincides with the beginning of the period that they will be needed for, therefore the lead time can be excluded from future calculation.

3.5.1.1 Silver-Meal Method

The purpose of Silver-Meal Heuristic method is to find step by them each and every reordering point, starting from the first one until the TRCUT is at a local minimum, at which point the system is reset, but this time the upcoming period becomes the first
one. By using this method and exploiting the following mathematical formula, 4 order points have been found (table 18):

\[
TRCUT(k) = \frac{C_p + \sum_{k=1}^{n}(k - 1)D(k) \times C_h}{k}
\]

Or, broken down step by step it goes as follows:

\[
TRCUT(k) = \frac{TRC(k)}{k} = \frac{A + \text{carrying costs}}{k}
\]

**For t=1:**

\[k=1\]
\[D=370\]
Therefore:

\[TRCUT(1) = \frac{C_p}{1} = \frac{38.75}{1} = 38.75\]

**For t=2:**

\[k=2\]
\[D=370\]
Therefore:

\[TRCUT(2) = \frac{C_p + D(2)C_h}{2} = \frac{38.75 + 370 \times 0.013}{2} = 21.84\]

**For t=3:**

\[k=3\]
D=161

Therefore:

\[
TRCUT(3) = \frac{C_p + D(2)C_h + 2D(3)C_h}{3} = \frac{38.75 + 370 \times 0.013 + 2 \times 161 \times 0.013}{3} = 15.99
\]

For t=4:

k=4

D=161

Therefore:

\[
TRCUT(4) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h}{4} = \frac{38.75 + 370 \times 0.013 + 2 \times 161 \times 0.013 + 3 \times 161 \times 0.013}{4} = 13.61
\]

For t=5:

k=5

D=100

Therefore:

\[
TRCUT(5) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h + 4D(5)C_h}{5} = \frac{38.75 + 370 \times 0.013 + 2 \times 161 \times 0.013 + 3 \times 161 \times 0.013 + 4 \times 100 \times 0.013}{5} = 11.95
\]

For t=6:
Therefore:

\[ TRCUT(6) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h + 4D(5)C_h + 5D(6)C_h}{6} \]

\[ = \frac{38.75 + 370 \cdot 0.013 + 2 \cdot 161 \cdot 0.013 + 3 \cdot 161 \cdot 0.013 + 4 \cdot 100 \cdot 0.013 + 5 \cdot 100 \cdot 0.013}{6} \]

\[ = 10.46 \]

For \( t=7 \):

\[ TRCUT(7) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h + 4D(5)C_h + 5D(6)C_h + 6D(7)C_h}{7} \]

\[ = \frac{38.75 + 370 \cdot 0.013 + 2 \cdot 161 \cdot 0.013 + 3 \cdot 161 \cdot 0.013 + 4 \cdot 100 \cdot 0.013 + 5 \cdot 100 \cdot 0.013 + 6 \cdot 85 \cdot 0.013}{7} \]

\[ = 10.46 \]

For \( t=8 \):

\[ TRCUT(8) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h + 4D(5)C_h + 5D(6)C_h + 6D(7)C_h + 7D(8)C_h}{8} \]

\[ = \frac{38.75 + 370 \cdot 0.013 + 2 \cdot 161 \cdot 0.013 + 3 \cdot 161 \cdot 0.013 + 4 \cdot 100 \cdot 0.013 + 5 \cdot 100 \cdot 0.013 + 6 \cdot 85 \cdot 0.013 + 7 \cdot 85 \cdot 0.013}{8} \]

\[ = 15.45 \]
At this point we can see that TRCUT(8) > TRCUT(7), therefore the algorithm terminates and we have our first replenishing point.

Now we rerun the algorithm, stating from $t=8$ but resetting $k$ to 1.

**For $t=8$:**

$k=1$

$D=85$

Therefore:

$$TRCUT(1) = \frac{C_p}{1} = \frac{38.75}{1} = 38.75$$

**For $t=9$:**

$k=2$

$D=123$

Therefore:

$$TRCUT(2) = \frac{C_p + D(2)C_h}{2} = \frac{38.75 + 123 \times 0.013}{2} = 20.20$$

**For $t=10$:**

$k=3$

$D=123$

Therefore:
Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse

\[ TRCUT(3) = \frac{C_p + D(2)C_h + 2D(3)C_h}{3} = \frac{38.75 + 123 \times 0.013 + 2 \times 123 \times 0.013}{3} = 14.56 \]

For \( t=11 \):

\( k=4 \)

\( D=113 \)

Therefore:

\[ TRCUT(4) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h}{4} = \frac{38.75 + 123 \times 0.013 + 2 \times 123 \times 0.013 + 3 \times 113 \times 0.013}{4} = 12.06 \]

For \( t=12 \):

\( k=5 \)

\( D=113 \)

Therefore:

\[ TRCUT(5) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h + 4D(5)C_h}{5} \]

\[ = \frac{38.75 + 123 \times 0.013 + 2 \times 123 \times 0.013 + 3 \times 113 \times 0.013 + 4 \times 113 \times 0.013}{5} = 10.85 \]

For \( t=13 \):

\( k=6 \)
D=93

Therefore:

\[
TRCUT(6) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h + 4D(5)C_h + 5D(6)C_h}{6} \\
= \frac{38.75 + 123 \times 0.013 + 2 \times 123 \times 0.013 + 3 \times 113 \times 0.013 + 4 \times 113 \times 0.013 + 5 \times 93 \times 0.013}{6} \\
= 10.08
\]

For t=14:

k=7
D=93

Therefore:

\[
TRCUT(7) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h + 4D(5)C_h + 5D(6)C_h + 6D(7)C_h}{7} \\
= \frac{38.75 + 123 \times 0.013 + 2 \times 123 \times 0.013 + 3 \times 113 \times 0.013 + 4 \times 113 \times 0.013 + 5 \times 93 \times 0.013 + 6 \times 93 \times 0.013}{7} \\
= 13.88
\]

At this point we can see that TRCUT(7) > TRCUT(6), therefore the algorithm terminates and we have our second replenishing point.

Now we rerun the algorithm, stating from t=14 but resetting k to 1.

For t=14:

k=1
D=93
Therefore:

\[ TRCUT(1) = \frac{C_p}{1} \]
\[ = \frac{38.75}{1} = 38.75 \]

For t=15:

k=2
D=83

Therefore:

\[ TRCUT(2) = \frac{C_p + D(2)C_h}{2} = \frac{38.75 + 83 \cdot 0.013}{2} = 19.93 \]

For t=16:

k=3
D=83

Therefore:

\[ TRCUT(3) = \frac{C_p + D(2)C_h + 2D(3)C_h}{3} = \frac{38.75 + 83 \cdot 0.013 + 2 \cdot 83 \cdot 0.013}{3} \]
\[ = 14.02 \]

For t=17:

k=4
D=429
Therefore:

\[
TRCUT(4) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h}{4} = \frac{38.75 + 83 \times 0.013 + 2 \times 83 \times 0.013 + 3 \times 429 \times 0.013}{4} = 19.93
\]

At this point we can see that TRCUT(4) > TRCUT(3), therefore the algorithm terminates and we have our third replenishing point.

Now we rerun the algorithm, stating from t=17 but resetting k to 1.

**For t=17:**

k=1
D=429
Therefore:

\[
TRCUT(1) = \frac{C_p}{1} = \frac{38.75}{1} = 38.75
\]

**For t=18:**

k=2
D=429
Therefore:
Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse

\[
TRCUT(2) = \frac{C_p + D(2)C_h}{2} = \frac{38.75 + 429 \times 0.013}{2} = 22.24
\]

For t=19:

\[k=3\]
\[D=406\]

Therefore:

\[
TRCUT(3) = \frac{C_p + D(2)C_h + 2D(3)C_h}{3} = \frac{38.75 + 429 \times 0.013 + 2 \times 406 \times 0.013}{3} = 18.43
\]

For t=20:

\[k=4\]
\[D=406\]

Therefore:

\[
TRCUT(4) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h}{4} = \frac{38.75 + 429 \times 0.013 + 2 \times 406 \times 0.013 + 3 \times 406 \times 0.013}{4} = 26.09
\]

At this point we can see that TRCUT(4) > TRCUT(3), therefore the algorithm terminates and we have our fourth replenishing point.

Now we rerun the algorithm, stating from t=20 but resetting k to 1.

For t=20:
k=1
D=406
Therefore:
\[ TRCUT(1) = \frac{C_p}{1} \]
\[ = \frac{38.75}{1} = 38.75 \]

For t=21:
k=2
D=391
Therefore:
\[ TRCUT(2) = \frac{C_p + D(2)C_h}{2} = \frac{38.75 + 391 \times 0.013}{2} = 21.98 \]

For t=22:
k=3
D=391
Therefore:
\[ TRCUT(3) = \frac{C_p + D(2)C_h + 2D(3)C_h}{3} = \frac{38.75 + 391 \times 0.013 + 2 \times 377 \times 0.013}{3} = 18.14 \]
Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse

For t=23:

k=4
D=377

Therefore:

\[
TRCUT(4) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h}{4} = \frac{38.75 + 391 \times 0.013 + 2 \times 377 \times 0.013 + 3 \times 377 \times 0.013}{4} = 25.06
\]

At this point we can see that TRCUT(4) > TRCUT(3), therefore the algorithm terminates and we have our fifth replenishing point.

Now we rerun the algorithm, starting from t=23 but resetting k to 1.

For t=23:

k=1
D=377

Therefore:

\[
TRCUT(1) = \frac{C_p}{1} = \frac{38.75}{1} = 38.75
\]

For t=24:

k=2
D=377
Therefore:

\[
TRCUT(2) = \frac{C_p + D(2)C_h}{2} = \frac{38.75 + 377 \times 0.013}{2} = 21.89
\]

**For t=25:**

k=3

D=370

Therefore:

\[
TRCUT(3) = \frac{C_p + D(2)C_h + 2D(3)C_h}{3} = \frac{38.75 + 377 \times 0.013 + 2 \times 370 \times 0.013}{3} = 17.88
\]

**For t=26:**

k=4

D=370

Therefore:

\[
TRCUT(4) = \frac{C_p + D(2)C_h + 2D(3)C_h + 3D(4)C_h}{4} = \frac{38.75 + 377 \times 0.013 + 2 \times 370 \times 0.013 + 3 \times 370 \times 0.013}{4} = 24.54
\]

At this point we can see that TRCUT(4) > TRCUT(3), therefore the algorithm terminates and we have our last replenishing point.
### SILVER-MEAL

<table>
<thead>
<tr>
<th>15 day period</th>
<th>t</th>
<th>Ft</th>
<th>TRCUT</th>
<th>Order Qty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>370</td>
<td>0.00</td>
<td>38.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>370</td>
<td>4.93</td>
<td>43.68</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>161</td>
<td>4.30</td>
<td>47.98</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>161</td>
<td>6.44</td>
<td>54.42</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>100</td>
<td>5.34</td>
<td>59.76</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>100</td>
<td>6.67</td>
<td>66.43</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>85</td>
<td>6.79</td>
<td>73.23</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>85</td>
<td>15.85</td>
<td>123.56</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>85</td>
<td>0.00</td>
<td>38.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>123</td>
<td>1.65</td>
<td>40.40</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>123</td>
<td>3.29</td>
<td>43.69</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>113</td>
<td>4.54</td>
<td>48.22</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>113</td>
<td>6.05</td>
<td>54.27</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>93</td>
<td>6.22</td>
<td>60.50</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>93</td>
<td>14.93</td>
<td>97.17</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>93</td>
<td>0.00</td>
<td>38.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>83</td>
<td>1.10</td>
<td>39.85</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>83</td>
<td>2.21</td>
<td>42.06</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>429</td>
<td>34.34</td>
<td>79.71</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>429</td>
<td>0.00</td>
<td>38.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>429</td>
<td>5.72</td>
<td>44.47</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>406</td>
<td>10.83</td>
<td>55.30</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>406</td>
<td>32.49</td>
<td>104.35</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>406</td>
<td>0.00</td>
<td>38.75</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>406</td>
<td>0.00</td>
<td>38.75</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>406</td>
<td>0.00</td>
<td>38.75</td>
</tr>
</tbody>
</table>

Table 19 Silver-Meal Table

Ch= 0.013 €/unit/15days

Cp= 38.75
The following order points have been calculated

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>T+L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order</td>
<td>1347</td>
<td>105</td>
</tr>
<tr>
<td>2nd order</td>
<td>652</td>
<td>90</td>
</tr>
<tr>
<td>3rd order</td>
<td>259</td>
<td>45</td>
</tr>
<tr>
<td>4th order</td>
<td>1265</td>
<td>45</td>
</tr>
<tr>
<td>5th order</td>
<td>1189</td>
<td>45</td>
</tr>
<tr>
<td>6th order</td>
<td>1125</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 20. Silver - Meal reorder points

A very important variable is the safety stock level, and when there are variations in demand then it's imperative to have sufficient safety stock in order to withstand the fluctuations in demand. Safety stock can be calculated by the following formula:

\[ SS = \sigma_{L+T} * z \]

Moreover the value of parameter S can be calculated from the following formula:
\[ S = \mu + z \times \sigma \]  \hspace{2cm} (23)

Where,

\( S \) is the maximum level of inventory

\( \mu \) is the average value of the available data for time interval T+L

\( \sigma \) is the standard deviation of the available data for time interval T+L

Using the calculated \( \mu(L+T) \) and \( \sigma(L+T) \) values from table 18, SS and \( S \) values can be calculated now but first, \( \phi(z) \) has to be determined:

1\(^{st}\) order:

\[
\phi(z) = 1 - \frac{Q^*_1 \times C_h}{R \times C_s} = 0.866
\]

Afterwards, \( z \) and \( H(z) \) can be derived from the natural distribution table. Therefore, \( z = 1.11 \) and \( H(z) = 0.0673 \)

Similarly, for Order 2 through 6, same procedure takes place until we have the full table shown below:

<table>
<thead>
<tr>
<th>Order</th>
<th>( Q )</th>
<th>( \phi(z) )</th>
<th>( z )</th>
<th>( H(z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order</td>
<td>1347</td>
<td>0.866</td>
<td>1.11</td>
<td>0.0673</td>
</tr>
<tr>
<td>2nd order</td>
<td>652</td>
<td>0.935</td>
<td>1.52</td>
<td>0.028</td>
</tr>
</tbody>
</table>

*Table 21 z and H(z) values of Silver Meal*
Now, in order to calculate the values of S and SS, the order point period values of μ(L+T) and σ(L+T) have to be calculated first, which are the average values of the 15-day instances of each period.

Therefore for 1st order it’s the average values of the first 7 15-day instances, for the second order the following 6 15-day instances and for the remaining four orders, it’s every next set of 3 15-day instances. These values are expressed on the following table.

<table>
<thead>
<tr>
<th>Order</th>
<th>μ(L+T)</th>
<th>σ(L+T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>192</td>
<td>69.86</td>
</tr>
<tr>
<td>2nd</td>
<td>109</td>
<td>39.43</td>
</tr>
<tr>
<td>3rd</td>
<td>86</td>
<td>31.30</td>
</tr>
<tr>
<td>4th</td>
<td>422</td>
<td>152.99</td>
</tr>
<tr>
<td>5th</td>
<td>396</td>
<td>143.85</td>
</tr>
<tr>
<td>6th</td>
<td>375</td>
<td>136.04</td>
</tr>
</tbody>
</table>

Now using the aforementioned formulas we come up with the final table of S and SS values,
Finally, after the calculation of $S$ and $SS$ for every time interval, using the known formulas 24 and 25 respectively, the total cost can be calculated with the following formula:

$$G(V, S) = C_p * V + C_h * \left( \frac{1}{6} \left( \sum_{i=1}^{V} SS_i + \sum_{i=1}^{V} \frac{S_i - SS_i}{2} \right) + \sum_{i=1}^{V} C_s * H(z) * \sigma_{L+T} \right)$$  \hspace{1cm} (32)$$

Where,

$V$ is the amount of orders placed and

Therefore,

$$G(V, S) = C_p * 6 + C_h * \left( \frac{1}{6} \left( \sum_{i=1}^{4} SS_i + \sum_{i=1}^{4} \frac{S_i - SS_i}{2} \right) + \sum_{i=1}^{4} C_s * H(z) * \sigma_{L+T} \right)$$  \hspace{1cm} (32)$$

$$G(T, S) = 316.43 \text{ €/year}$$
Which is higher than the current system results of 286€/year.

3.5.1.2 Part-Period Balancing

The procedure for the PPB method has been explained in 2.6.4 and now it will be used to find the order quantity for our data set by calculating the carrying cost and then picking the period at which the carrying cost is closer to the fixed order cost.

\[ CarryingCost(k) = \sum_{k=1}^{n} (k - 1)D(k) \cdot C_h \]

For \( t=1 \):

\[ k=1 \]
\[ D=370 \]

Therefore:

\[ CarryingCost(k) = \sum_{k=1}^{n} (1 - 1)D(k) \cdot C_h = 0 \]

And

\[ |CarryingCost(1) - C_p| = 38.75 \]

For \( t=2 \):

\[ k=2 \]
\[ D=370 \]

Therefore:
CarryingCost(2) = D(2) * C_h = 370 * 0.013 = 4.93

And

\[ |CarryingCost(2) - C_p| = |4.93 - C_p| = 33.82 \]

For t=3:

k=3
D=161
Therefore:

CarryingCost(3) = D(2) * C_h + 2D(3) * C_h = 370 * 0.013 + 2 * 161 * 0.013 = 9.23

And

\[ |CarryingCost(3) - C_p| = |9.23 - C_p| = 29.52 \]

For t=4:

k=4
D=161
Therefore:

CarryingCost(4) = D(2) * C_h + 2D(3) * C_h + 3D(4) * C_h
= 370 * 0.013 + 2 * 161 * 0.013 + 3 * 161 * 0.013 = 15.67

And
Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse

\[ |CarryingCost(4) - C_p| = |15.67 - C_p| = 23.08 \]

**For t=5:**

\[ k=5 \]

\[ D=100 \]

Therefore:

\[
CarryingCost(5) = D(2) \cdot C_h + 2D(3) \cdot C_h + 3D(4) \cdot C_h + 4D(5) \cdot C_h \\
= 370 \cdot 0.013 + 2 \cdot 161 \cdot 0.013 + 3 \cdot 161 \cdot 0.013 + 4 \cdot 100 \cdot 0.013 \\
= 21.01
\]

And

\[ |CarryingCost(5) - C_p| = |21.01 - C_p| = 17.74 \]

**For t=6:**

\[ k=6 \]

\[ D=100 \]

Therefore:

\[
CarryingCost(6) \\
= D(2) \cdot C_h + 2D(3) \cdot C_h + 3D(4) \cdot C_h + 4D(5) \cdot C_h + 5D(6) \cdot C_h \\
= 370 \cdot 0.013 + 2 \cdot 161 \cdot 0.013 + 3 \cdot 161 \cdot 0.013 + 4 \cdot 100 \cdot 0.013 + 5 \cdot 100 \cdot 0.013 \\
= 27.68
\]
And

\[ |CarryingCost(6) - C_p| = |27.68 - C_p| = 11.07 \]

For \( t=7 \):

\( k=7 \)

\( D=85 \)

Therefore:

\[
CarryingCost(7) = D(2) * C_h + 2D(3) * C_h + 3D(4) * C_h + 4D(5) * C_h + 5D(6) * C_h + 6D(7) * C_h
\]

\[
= 370 * 0.013 + 2 * 161 * 0.013 + 3 * 161 * 0.013 + 4 * 100 * 0.013 + 5 * 100 * 0.013 + 6 * 85 * 0.013 = 34.48
\]

And

\[ |CarryingCost(7) - C_p| = |34.48 - C_p| = 4.27 \]

For \( t=8 \):

\( k=8 \)

\( D=85 \)

Therefore:
CarryingCost(8)

\[= D(2) \cdot C_h + 2D(3) \cdot C_h + 3D(4) \cdot C_h + 4D(5) \cdot C_h + 5D(6) \cdot C_h + 6D(7) \cdot C_h + 7D(8) \cdot C_h \]

\[= 370 \cdot 0.013 + 2 \cdot 161 \cdot 0.013 + 3 \cdot 161 \cdot 0.013 + 4 \cdot 100 \cdot 0.013 + 5 \cdot 100 \cdot 0.013 + 6 \cdot 85 \cdot 0.013 + 7 \cdot 85 \cdot 0.013 + 8 \cdot 123 \cdot 0.013 = 42.41 \]

And

\[|CarryingCost(8) - C_p| = |42.41 - C_p| = 3.66 \]

For t=9:

k=9

D=123

Therefore:

CarryingCost(9)

\[= D(2) \cdot C_h + 2D(3) \cdot C_h + 3D(4) \cdot C_h + 4D(5) \cdot C_h + 5D(6) \cdot C_h + 6D(7) \cdot C_h + 7D(8) \cdot C_h + 8D(9) \cdot C_h \]

\[= 370 \cdot 0.013 + 2 \cdot 161 \cdot 0.013 + 3 \cdot 161 \cdot 0.013 + 4 \cdot 100 \cdot 0.013 + 5 \cdot 100 \cdot 0.013 + 6 \cdot 85 \cdot 0.013 + 7 \cdot 85 \cdot 0.013 + 8 \cdot 123 \cdot 0.013 + 9 \cdot 123 \cdot 0.013 = 55.57 \]

And

\[|CarryingCost(9) - C_p| = |55.57 - C_p| = 16.82 \]
But at this stage, 
\[ |CarryingCost(7) - C_p| > |CarryingCost(8) - C_p| < |CarryingCost(9) - C_p| \]
, therefore the algorithm breaks and the order point is set for \( t=8 \), which brings the total order quantity at \( Q_1^* = 1432 \).

Now, the algorithm is reset and it starts again for \( t = 9 \).

**For t=9:**

\( k = 1 \)

\( D = 123 \)

Therefore:

\[ CarryingCost(k) = \sum_{k=1}^{n} (1 - 1)D(k) \ast C_h = 0 \]

And

\[ |CarryingCost(1) - C_p| = 38.75 \]

**For t=10:**

\( k = 2 \)

\( D = 123 \)

Therefore:

\[ CarryingCost(2) = D(2) \ast C_h = 123 \ast 0.013 = 1.65 \]
And

$$|CarryingCost(2) - C_p| = |1.65 - C_p| = 37.10$$

**For t=11:**

k=3

D=113

Therefore:

$$CarryingCost(3) = D(2) * C_h + 2D(3) * C_h = 123 * 0.013 + 2 * 113 * 0.013 = 4.67$$

And

$$|CarryingCost(3) - C_p| = |4.67 - C_p| = 34.08$$

**For t=12:**

k=4

D=113

Therefore:

$$CarryingCost(4) = D(2) * C_h + 2D(3) * C_h + 3D(4) * C_h$$

$$= 123 * 0.013 + 2 * 113 * 0.013 + 3 * 113 * 0.013 = 9.21$$

And

$$|CarryingCost(4) - C_p| = |9.21 - C_p| = 29.54$$
For $t=13$:

$k=5$

$D=93$

Therefore:

$$\text{CarryingCost}(5) = D(2) * C_h + 2D(3) * C_h + 3D(4) * C_h + 4D(5) * C_h$$

$$= 123 * 0.013 + 2 * 113 * 0.013 + 3 * 113 * 0.013 + 4 * 93 * 0.013$$

$$= 14.18$$

And

$$|\text{CarryingCost}(5) - C_p| = |24.57 - C_p| = 24.57$$

For $t=14$:

$k=6$

$D=93$

Therefore:

$$\text{CarryingCost}(6) = D(2) * C_h + 2D(3) * C_h + 3D(4) * C_h + 4D(5) * C_h + 5D(6) * C_h$$

$$= 123 * 0.013 + 2 * 113 * 0.013 + 3 * 113 * 0.013 + 4 * 93 * 0.013$$

$$+ 5 * 93 * 0.013 = 20.41$$

And
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\[ |CarryingCost(6) - C_p| = |20.41 - C_p| = 18.34 \]

**For t=15:**

k=7  
D=83  
Therefore:

\[
CarryingCost(7) = D(2) \cdot C_h + 2D(3) \cdot C_h + 3D(4) \cdot C_h + 4D(5) \cdot C_h + 5D(6) \cdot C_h + 6D(7) \cdot C_h \\
= 123 \cdot 0.013 + 2 \cdot 113 \cdot 0.013 + 3 \cdot 113 \cdot 0.013 + 4 \cdot 93 \cdot 0.013 + 5 \cdot 93 \cdot 0.013 + 6 \cdot 83 \cdot 0.013 = 27.02
\]

And

\[ |CarryingCost(7) - C_p| = |27.02 - C_p| = 11.73 \]

**For t=16:**

k=8  
D=83  
Therefore:

\[
CarryingCost(8) = D(2) \cdot C_h + 2D(3) \cdot C_h + 3D(4) \cdot C_h + 4D(5) \cdot C_h + 5D(6) \cdot C_h + 6D(7) \cdot C_h + 7D(8) \cdot C_h \\
= 123 \cdot 0.013 + 2 \cdot 113 \cdot 0.013 + 3 \cdot 113 \cdot 0.013 + 4 \cdot 93 \cdot 0.013 + 5 \cdot 93 \cdot 0.013 + 6 \cdot 83 \cdot 0.013 + 7 \cdot 83 \cdot 0.013 = 34.74
\]
And

\[ |CarryingCost(8) - C_p| = |34.74 - C_p| = 4.01 \]

For \( t=17 \):

\( k=9 \)

\( D=429 \)

Therefore:

\[ CarryingCost(9) \]

\[ = D(2) \times C_h + 2D(3) \times C_h + 3D(4) \times C_h + 4D(5) \times C_h + 5D(6) \times C_h + 6D(7) \times C_h + 7D(8) \times C_h + 8D(9) \times C_h \]

\[ = 123 \times 0.013 + 2 \times 113 \times 0.013 + 3 \times 113 \times 0.013 + 4 \times 93 \times 0.013 + 5 \times 93 \times 0.013 + 6 \times 83 \times 0.013 + 7 \times 83 \times 0.013 + 8 \times 123 \times 0.013 \]

\[ = 80.53 \]

And

\[ |CarryingCost(9) - C_p| = |80.53 - C_p| = 41.78 \]

But at this stage,

\[ |CarryingCost(7) - C_p| > |CarryingCost(8) - C_p| < |CarryingCost(9) - C_p| \]

, therefore the algorithm breaks and the order point is set for \( t=8 \), which brings the total order quantity at \( Q_2^* = 826 \)

Now, the algorithm is reset and it starts again for \( t = 17 \).
For t=17:

\( k=1 \)

\( D=429 \)

Therefore:

\[ CarryingCost(k) = \sum_{k=1}^{n} (1 - 1)D(k) \cdot C_h = 0 \]

And

\[ |CarryingCost(1) - C_p| = 38.75 \]

For t=18:

\( k=2 \)

\( D=429 \)

Therefore:

\[ CarryingCost(2) = D(2) \cdot C_h = 429 \cdot 0.013 = 5.72 \]

And

\[ |CarryingCost(2) - C_p| = |5.72 - C_p| = 33.03 \]

For t=19:

\( k=3 \)

\( D=406 \)

Therefore:
\[ CarryingCost(3) = D(2) \cdot C_h + 2D(3) \cdot C_h = 429 \cdot 0.013 + 2 \cdot 406 \cdot 0.013 \]
\[ = 16.55 \]

And

\[ |CarryingCost(3) - C_p| = |16.55 - C_p| = 22.2 \]

**For t=20:**

k=4

D=406

Therefore:

\[ CarryingCost(4) = D(2) \cdot C_h + 2D(3) \cdot C_h + 3D(4) \cdot C_h \]
\[ = 429 \cdot 0.013 + 2 \cdot 406 \cdot 0.013 + 3 \cdot 406 \cdot 0.013 = 32.8 \]

And

\[ |CarryingCost(4) - C_p| = |32.8 - C_p| = 5.95 \]

**For t=21:**

k=5

D=391
Therefore:

\[
CarryingCost(5) = D(2) \times C_h + 2D(3) \times C_h + 3D(4) \times C_h + 4D(5) \times C_h \\
= 429 \times 0.013 + 2 \times 406 \times 0.013 + 3 \times 406 \times 0.013 + 4 \times 93 \times 0.013 \\
= 53.68
\]

And

\[
|CarryingCost(5) - C_p| = |53.68 - C_p| = 14.93
\]

But at this stage,

\[
|CarryingCost(4) - C_p| > |CarryingCost(5) - C_p| < |CarryingCost(6) - C_p|
\]

, therefore the algorithm breaks and the order point is set for \( t=4 \), which brings the total order quantity at \( Q_3^* = 1671 \)

Now, the algorithm resets and it starts again.

**For \( t=21 \):**

\( k=1 \)

\( D=391 \)

Therefore:

\[
CarryingCost(k) = \sum_{k=1}^{n} (1 - 1)D(k) \times C_h = 0
\]

And
$$|CarryingCost(1) - C_p| = 38.75$$

For $t=22$:

k=2

D=391

Therefore:

$$CarryingCost(2) = D(2) \times C_h = 391 \times 0.013 = 5.22$$

And

$$|CarryingCost(2) - C_p| = |5.22 - C_p| = 33.53$$

For $t=23$:

k=3

D=377

Therefore:

$$CarryingCost(3) = D(2) \times C_h + 2D(3) \times C_h = 391 \times 0.013 + 2 \times 377 \times 0.013$$

$$= 15.28$$

And

$$|CarryingCost(3) - C_p| = |15.28 - C_p| = 23.47$$

For $t=24$:

k=4
D=377

Therefore:

$$CarryingCost(4) = D(2) * C_h + 2D(3) * C_h + 3D(4) * C_h$$
$$= 391 * 0.013 + 2 * 377 * 0.013 + 3 * 377 * 0.013 = 30.37$$

And

$$|CarryingCost(4) - C_p| = |30.37 - C_p| = 8.38$$

**For t=25:**

\(k=5\)

D=370

Therefore:

$$CarryingCost(5) = D(2) * C_h + 2D(3) * C_h + 3D(4) * C_h + 4D(5) * C_h$$
$$= 391 * 0.013 + 2 * 377 * 0.013 + 3 * 377 * 0.013 + 4 * 93 * 0.013$$
$$= 50.11$$

And

$$|CarryingCost(5) - C_p| = |50.11 - C_p| = 11.36$$

But at this stage,

$$|CarryingCost(4) - C_p| > |CarryingCost(5) - C_p| < |CarryingCost(6) - C_p|$$
Therefore the algorithm breaks and the order point is set for $t=4$, which brings the total order quantity at $Q_4^*=1537$

So, the algorithm now terminates for good as it has covered all our periods and also closed the last order cycle successfully.

**Table 22 PPB Table**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ft</td>
<td>Carrying Cost</td>
<td>$</td>
</tr>
<tr>
<td>1</td>
<td>370</td>
<td>0</td>
<td>38.75</td>
</tr>
<tr>
<td>2</td>
<td>370</td>
<td>4.93</td>
<td>4.93</td>
</tr>
<tr>
<td>3</td>
<td>161</td>
<td>4.30</td>
<td>9.23</td>
</tr>
<tr>
<td>4</td>
<td>161</td>
<td>6.44</td>
<td>15.67</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>5.34</td>
<td>21.01</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>6.67</td>
<td>27.68</td>
</tr>
<tr>
<td>7</td>
<td>85</td>
<td>6.79</td>
<td>34.48</td>
</tr>
<tr>
<td>8</td>
<td>85</td>
<td>7.93</td>
<td>42.41</td>
</tr>
<tr>
<td>9</td>
<td>123</td>
<td>13.16</td>
<td>55.57</td>
</tr>
<tr>
<td>1</td>
<td>123</td>
<td>0</td>
<td>38.75</td>
</tr>
<tr>
<td>2</td>
<td>123</td>
<td>1.65</td>
<td>1.65</td>
</tr>
<tr>
<td>3</td>
<td>113</td>
<td>3.02</td>
<td>4.67</td>
</tr>
<tr>
<td>4</td>
<td>113</td>
<td>4.54</td>
<td>9.21</td>
</tr>
<tr>
<td>5</td>
<td>93</td>
<td>4.98</td>
<td>14.18</td>
</tr>
</tbody>
</table>
Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>93</td>
<td>6.22</td>
<td>20.41</td>
<td>18.34</td>
</tr>
<tr>
<td>7</td>
<td>83</td>
<td>6.62</td>
<td>27.02</td>
<td>11.73</td>
</tr>
<tr>
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<td>83</td>
<td>7.72</td>
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<td>4.01</td>
</tr>
<tr>
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<td>429</td>
<td>45.79</td>
<td>80.53</td>
<td>41.78</td>
</tr>
<tr>
<td>1</td>
<td>429</td>
<td></td>
<td></td>
<td>38.75</td>
</tr>
<tr>
<td>2</td>
<td>429</td>
<td>5.72</td>
<td>5.72</td>
<td>33.03</td>
</tr>
<tr>
<td>3</td>
<td>406</td>
<td>10.83</td>
<td>16.55</td>
<td>22.20</td>
</tr>
<tr>
<td>4</td>
<td>406</td>
<td>16.25</td>
<td>32.80</td>
<td>5.95</td>
</tr>
<tr>
<td>5</td>
<td>391</td>
<td>20.88</td>
<td>53.68</td>
<td>14.93</td>
</tr>
<tr>
<td>1</td>
<td>391</td>
<td></td>
<td></td>
<td>38.75</td>
</tr>
<tr>
<td>2</td>
<td>391</td>
<td>5.22</td>
<td>5.22</td>
<td>33.53</td>
</tr>
<tr>
<td>3</td>
<td>377</td>
<td>10.06</td>
<td>15.28</td>
<td>23.47</td>
</tr>
<tr>
<td>4</td>
<td>377</td>
<td>15.09</td>
<td>30.37</td>
<td>8.38</td>
</tr>
<tr>
<td>5</td>
<td>370</td>
<td>19.74</td>
<td>50.11</td>
<td>11.36</td>
</tr>
</tbody>
</table>

For all 4 break points (highlighted yellow) the following is true:

\[
|\text{CarryingCost}(T - 1) - C_p| > |\text{CarryingCost}(T) - C_p| \\
< |\text{CarryingCost}(T + 1) - C_p|
\]

The following table shows the ordering points and the order quantity.

<table>
<thead>
<tr>
<th>PPB</th>
<th>Q</th>
<th>T</th>
<th>T+L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order</td>
<td>1432</td>
<td>90</td>
<td>150</td>
</tr>
<tr>
<td>2nd order</td>
<td>826</td>
<td>90</td>
<td>150</td>
</tr>
<tr>
<td>3rd order</td>
<td>1671</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>4th order</td>
<td>1537</td>
<td>60</td>
<td>90</td>
</tr>
</tbody>
</table>

*Table 23 Part-Period Balancing reordering points*
Exactly like the procedure followed for the Silver-Meal method, we continue with the new data derived from PPB method.

1st order:

\[ \phi(z) = 1 - \frac{Q_1^+ \cdot C_h}{R \cdot C_s} = 0.848 \]

Afterwards, z and H(z) can be derived from the natural distribution table. Therefore,

z = 1.03 and 
H(z) = 0.0787

Similarly, for Orders 2 through 4, same procedure takes place until we have the full table shown bellow:

<table>
<thead>
<tr>
<th>Order</th>
<th>Q</th>
<th>(\phi(z))</th>
<th>z</th>
<th>H(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order</td>
<td>1432</td>
<td>0.848</td>
<td>1.03</td>
<td>0.0787</td>
</tr>
<tr>
<td>2nd order</td>
<td>826</td>
<td>0.912</td>
<td>1.36</td>
<td>0.04</td>
</tr>
<tr>
<td>3rd order</td>
<td>1671</td>
<td>0.822</td>
<td>0.93</td>
<td>0.095</td>
</tr>
<tr>
<td>4th order</td>
<td>1537</td>
<td>0.836</td>
<td>0.98</td>
<td>0.0865</td>
</tr>
</tbody>
</table>

Now, in order to calculate the values of S and SS, the order point period values of \(\mu(L+T)\) and \(\sigma(L+T)\) have to be calculated first, which are the average values of the 15day instances of each period.
Quantitative analysis of inventory Management System - Case study of an Auto-Parts Warehouse

Therefore for 1st order it’s the average values of the first 8 15-day instances, for the second order the following 8 15-day instances and for the remaining two orders, it’s every next set of 4 15-day instances. These values are expressed on the following table.

<table>
<thead>
<tr>
<th>Order</th>
<th>( \mu(L+T) )</th>
<th>( \sigma(L+T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order</td>
<td>179</td>
<td>64.98</td>
</tr>
<tr>
<td>2nd order</td>
<td>103</td>
<td>37.46</td>
</tr>
<tr>
<td>3rd order</td>
<td>418</td>
<td>151.59</td>
</tr>
<tr>
<td>4th order</td>
<td>384</td>
<td>139.49</td>
</tr>
</tbody>
</table>

Now using the aforementioned formulas we come up with the final table of \( S \) and \( SS \) values,

<table>
<thead>
<tr>
<th>Order #</th>
<th>( Cs \times H(z) \times \sigma L+T )</th>
<th>( S )</th>
<th>( SS )</th>
<th>( (S-SS)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.81265</td>
<td>245.97995</td>
<td>66.92918347</td>
<td>89.52538</td>
</tr>
<tr>
<td>2</td>
<td>0.824109</td>
<td>154.16397</td>
<td>50.94492027</td>
<td>51.60953</td>
</tr>
<tr>
<td>3</td>
<td>7.920721</td>
<td>558.69266</td>
<td>140.9812622</td>
<td>208.8557</td>
</tr>
<tr>
<td>4</td>
<td>6.6364</td>
<td>521.07548</td>
<td>136.7035636</td>
<td>192.186</td>
</tr>
<tr>
<td>sum=</td>
<td>18.19388</td>
<td>sum(SS)=</td>
<td>sum((S-SS)/2)=</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>395.5589296</td>
<td>542.1766</td>
<td></td>
</tr>
</tbody>
</table>

Finally, after the calculation of \( S \) and \( SS \) for every time interval, the total cost can be calculated with the following formula:

\[
G(V, S) = C_p \times V + C_h \times \frac{1}{4} \left( \sum_{i=1}^{V} SS_i + \sum_{i=1}^{V} \frac{S_i - SS_i}{2} \right) + \sum_{i=1}^{V} C_s \times H(z) \times \sigma_{L+T} \quad (32)
\]

Where,
V is the amount of orders placed and

Therefore,

\[
G(V, S) = C_p \times 4 + C_h \times \frac{1}{4} \left( \sum_{i=1}^{4} SS_i + \sum_{i=1}^{4} S_l \frac{S_l - SS_i}{2} \right) + \sum_{i=1}^{4} C_s \times H(z) \times \sigma_{L+T} \tag{32}
\]

\[
G(V, S) = 248.21 \text{ €/year}
\]

Which is lower than the current system results of 286€/year.

### 3.5.1.3 Economic Order Quantity

For the method of EOQ, we calculate EOQ with the following formula:

\[
EOQ = \sqrt{\frac{2 \times C_p \times R}{C_h}}
\]

Where,

C\(_p\) is the acquisition cost of the SKU

R is the annual demand which has been calculated by estimating the average of the annual demand for the period 2018

C\(_h\) and C\(_p\) are the holding and fixed order cost which have been addressed earlier.

Solving for EOQ:

\[
EOQ = 1150.615
\]

Which is approximately **1151 units**.
With $T$ approximating to 51 days there must be placed approximately 7.15 orders in a year following the EOQ method.

$\sigma$, $\phi(z)$, $z$ and $H(z)$ are similarly calculated as with the Silver-Meal method and $S$ and $SS$ can now also be found by their corresponding formulas which lead to the total cost formula:

$$\phi(z) = 1 - \frac{EOQ \times C_h}{R \times C_s} = 0.953$$

Therefore, from the normal distribution table, $z$ value and $H(z)$ become known:

<table>
<thead>
<tr>
<th>Q</th>
<th>$\phi(z)$</th>
<th>$z$</th>
<th>$H(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOQ</td>
<td>1151</td>
<td>0.953</td>
<td>1.68</td>
</tr>
</tbody>
</table>

From the following table, we were able to determine the ordering points for the EOQ method which will help us determine the exact length of the periods.

| 15-day period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | Total |
|---------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| **Starting Inventory** | 0 | 781 | 411 | 250 | 89 | 1140 | 1039 | 955 | 870 | 746 | 623 | 509 | 396 | 303 | 209 | 127 | 44 | 766 | 336 | 1081 | 675 | 284 | 1043 | 666 | **13628** |
| **Replenishment** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **1151** | **5755** |
| **Requirements** | 370 | 370 | 161 | 161 | 100 | 100 | 85 | 85 | 123 | 123 | 113 | 113 | 93 | 93 | 83 | 83 | 429 | 429 | 406 | 406 | 391 | 391 | 377 | 377 | **5466** |
| **Ending Inventory** | 781 | 411 | 250 | 89 | 1140 | 1039 | 955 | 870 | 746 | 623 | 509 | 396 | 303 | 209 | 127 | 44 | 766 | 336 | 1081 | 675 | 284 | 1043 | 666 | **289** | **13628** |

Therefore, order #1 will cover periods 1-4, order #2 will cover periods 5-16, order #3 will cover periods 17&18, order 34 will cover periods 19-21 and finally order #5 will cover periods 22-24.

Similarly with the other two methods, we can calculate values of $\mu(L+T)$ and $\sigma(L+T)$.
And of course values of $S$ and $SS$:

<table>
<thead>
<tr>
<th>Order #1</th>
<th>$Cs<em>H(z)</em>\sigma_{L+T}$</th>
<th>$S$</th>
<th>$SS$</th>
<th>$(S-SS)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.81265</td>
<td>245.97995</td>
<td>66.92918347</td>
<td>89.52538</td>
</tr>
<tr>
<td>2</td>
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<td>154.16397</td>
<td>50.94492027</td>
<td>51.60953</td>
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<td>3</td>
<td>7.920721</td>
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<td>140.9812622</td>
<td>208.8557</td>
</tr>
<tr>
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<td>521.07548</td>
<td>136.7035636</td>
<td>192.186</td>
</tr>
<tr>
<td>sum=</td>
<td>18.19388</td>
<td>sum(SS)= 395.5589296</td>
<td>sum((S-SS)/2)= 542.1766</td>
<td></td>
</tr>
</tbody>
</table>

Finally, the total cost can be calculated with the following formula since every variable is now known:

$$G(V, S) = C_p * 5 + C_h * \left( \sum_{i=1}^{5} SS_i + \sum_{i=1}^{5} \frac{S_i - SS_i}{2} \right) + \sum_{i=1}^{5} C_o * H(z) * \sigma_{L+T}$$

But the above cost is for 14 periods. So, we need to calculate the annual cost out of it. Therefore,

$$G(V, S) = 311.85 \text{€/year}$$
3.5.1.4 Comparison

Finally, of the 3 methods that have been used for the calculation of the total cost, Part-Period Balancing method proved to be the most efficient.

<table>
<thead>
<tr>
<th>Method</th>
<th>Total Annual Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver-Meal</td>
<td>316.43</td>
</tr>
<tr>
<td>PPB</td>
<td>248.21</td>
</tr>
<tr>
<td>EOQ</td>
<td>311.85</td>
</tr>
<tr>
<td>Firm</td>
<td>286.00</td>
</tr>
</tbody>
</table>
4 Conclusions

The past 3 years has been a period of drastic changes in the field of inventory management practices for Drakoglou which mostly consisted of digitization and automation of various inventory operations, while the warehouse has also seen an expansion of its capacity. All the above have been implemented by the personnel who had no substantial expertise in this particular field, therefore there are clearly some improvements that can take place.

Regarding the SKU that has been investigated for the purpose if this master thesis, the firm is using the excel function to forecast the demand which, in comparison to the forecast model presented here the divergence from the actual demand can be clearly witnessed (figure 11). The proposed model seems to be more in touch with the actual demand throughout the forecasted period.

![Figure 11: Comparison between Firm’s forecast and proposed forecast](image-url)
Regarding the periodic review policy of the inventory system, three methods have been studied during this thesis which showed slightly different results. Concerning the optimal time interval between two successive orders, three methods have been investigated with varying results, both in order frequency and in total cost. Out of the three, PPB had the best result, bringing down the annual expenses from the current state of 286 to 248,21. On the other hand, both EOQ and Silver-Meal failed to better the current system used by the firm.

The data set that was available to us had some peculiarities and that would be probably because there were 2 SKU’s combined in one and during the period that one was replacing the other, there might have been some abnormal discrepancies in the demand scheme or even in the supply chain part of the company, failing to promote the new product, placing false orders or mixing up the inventory levels between the two codes.
All in all, in the future the model should be revisited in order to witness whether it is successful or not and make the necessary changes that will make it more accurate and usable for the company.
5 Bibliography


