ΥΠΟΛΟΓΙΣΤΙΚΗ ΑΝΑΛΥΣΗ ΣΥΓΧΡΟΝΩΝ ΠΡΟΤΥΠΩΝ ΣΤΟΙΧΕΙΩΔΩΝ ΣΩΜΑΤΙΔΙΩΝ ΚΑΙ ΚΟΣΜΟΛΟΓΙΑΣ
CALCULATIONAL ANALYSIS OF ELEMENTARY PARTICLES AND COSMOLOGY MODERN MODELS

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1 Introduction

The present thesis is based on study and research that resulted in two novel baryogenesis models and an investigation of the neutrinos mixing matrix by elements of the $PSL(2,7)$ finite group. The relative publications are


Chapter 1 of the thesis is the introduction. Chapter 2 is about baryogenesis, its requirements and categories of baryogenesis models.

In chapter 3 the Randall-Sundrum II gravity model is presented, along with its cosmology and the characteristics of primordial black holes in the frame of this model.

In chapter 4 a novel cosmological scenario, capable to generate the observed baryon number at the electroweak scale for very small CP-violating angles, is presented. The proposed mechanism can be applied in conventional FRW cosmology but becomes extremely efficient due to accretion in the context of early cosmic expansion with high energy modifications. Assuming that our universe is a Randall-Sundrum brane, baryon asymmetry can easily be produced by Hawking radiation of very small primordial black holes. The Hawking radiation reheats a spherical region around every black hole to a high temperature and the electroweak symmetry is restored there. A domain wall is formed separating the region with the symmetric vacuum from the asymmetric region. Electroweak baryogenesis takes place in the domain wall. First order phase transition is not needed. The black holes’ lifetime is prolonged due to accretion, resulting in strong efficiency of the baryon producing mechanism. The allowed by the mechanism black hole mass range includes masses that are energetically favoured to be produced from interactions around the higher dimensional Planck scale.

Chapter 5 is a presentation of Brans-Dicke modified gravity, its cosmology and primordial black holes in a Brans-Dicke universe.
In chapter 6 we apply the baryogenesis mechanism presented in chapter 4 in the cosmological frame of Brans-Dicke modified gravity. Very small primordial black holes are produced at the end of the Bran-Dicke field domination era. We show that the accretion by the black holes can be strong enough to lead to black holes’ domination. Thus the observed baryon number is produced by the black holes for a CP-violation angle that is predicted by a 2-Higgs doublet model.

Chapter 7 is a brief introduction to the problem of neutrino masses and mixing. Symmetries of the mass matrix based on discrete groups that can explain the observed patterns are also discussed.

In chapter 8 three-dimensional unitary representations of the projective linear group $PSL_2(7)$ are derived. Based on the observation that the generators of the group exhibit a latin square pattern, we use available computational packages on discrete algebra to determine the generic properties of the group elements. We present analytical expressions and discuss several examples which reproduce the neutrino mixing angles in accordance with the experimental data.
2 Baryogenesis

2.1 Observation of the baryon asymmetry

Baryogenesis is a key question of cosmology and is still open. It refers to the dominance of matter over antimatter in the universe. Antiparticles have almost identical properties with particles, so we would expect to be produced in equal numbers during the first stages of the universe. Clearly, this is not the case. The existence of antimatter in large numbers would lead to the production of high energy electromagnetic radiation due to matter-antimatter annihilation. This is not observed.

More specifically, $p - \bar{p}$ annihilations create pions. Then, $\pi^0$ decay into high energy photons. The lack of excessive radiation at the observations of 100 MeV cosmic $\gamma$-rays set quite strict limits:

- if antigalaxies exist, they could be no closer than $\simeq 10 Mpc$ [4],
- the antimatter to matter ratio in the observed collision of two galaxies of the Bullet Cluster is less than $3 \times 10^{-6}$ [5],
- the antistars antibaryon to stars baryon ratio in our galaxy is smaller than $4 \cdot 10^{-5}$ within 150 pc from the sun [6].

Traces of antibaryons are found in the cosmic radiation with an antiproton to proton ratio equal to $10^{-4}$, but this is a secondary product of baryons, leptons and photons collisions in the interstellar medium. In figure 1 it is shown the accordance of the observed antiprotons with the calculation of the production as collisions of baryons, leptons or photons in the interstellar medium.

The amount of baryons in the universe has been calculated from BBN (bing bang nucleosynthesis) and CMB (cosmic microwave background) data, assuming a negligible antimatter amount. It is remarkable that both methods give very similar results, despite the fact that BBN measures the universe baryon amount at an age of 100 sec, while CMB measures it at an age of $\sim 370,000$ yrs.

The nuclei of the lighter elements were produced during BBN. $^4 He$ measurement gives [9]

$$\Omega_b h^2 = 0.0234 \pm 0.0019$$

(1)

where $\Omega_b$ is the ratio of the universe’s baryon density to the critical density $\rho_c$. 

5
The amount of primordial deuterium is also used and the accuracy is better by one order of magnitude. With $D/H = (2.53 \pm 0.04) \cdot 10^{-5}$ \[10\], it is calculated that

$$\Omega_b h^2 = 0.02202 \pm 0.00045.$$  \hspace{1cm} \text{(2)}

The most accurate method of measuring $\Omega_b$ is by using the CMB’s angular fluctuations data. The first acoustic peak of the anisotropy power spectrum of CMB is very sensitive to the number of baryons at the epoch of the decoupling of matter and radiation (figure 2).

Using the measurements from the Planck satellite, the amount of baryons is calculated to be \[11\]

$$\Omega_b h^2 = 0.02205 \pm 0.00028.$$  \hspace{1cm} \text{(3)}

$h \simeq 0.67$, so the above value of $\Omega_b h^2$ is equivalent to

$$\Omega_b = 0.049 \pm 0.0007.$$  \hspace{1cm} \text{(4)}

This $\Omega_b$ value means that the universe contains $\sim 5\%$ baryons. Note that the directly observed baryon energy density of galaxies is only about 10\% of the estimated baryon amount. A very recent development is the detection of a large portion of the missing baryon amount in the form of ionized gas filaments linking neighboring galaxies \[13, 14\].

The baryon asymmetry is often expressed as the ratio of the baryon-antibaryon number density difference to the CMB photon number density. According to the data mentioned
Figure 2: The sensitivity of the first acoustic peak in the Cosmic Microwave Background to the average baryon density of the Universe. (Taken from [12]).

above, it is

\[ n = \frac{n_b - n_b}{n_\gamma} = (6.1 \pm 0.3) \cdot 10^{-10}. \]  \hfill (5)

### 2.2 Requirements for baryogenesis

One could accept the baryon imbalance existing from the beginning of the universe. This is though not very natural. The other option is the creation of equal quantities of matter and anti-matter. Then, matter should dominate over antimatter via some physical processes. The standard mechanism of baryogenesis demands the concurrent satisfaction of three physical requirements, as it was first explained by Sakharov [21]:

1. Baryon non-conserving processes. This can be achieved either at the grand unification scale [22] or even at the electroweak energy scale [23].
2. C and CP-violation which has already been observed in the experiment.
3. Out of equilibrium conditions. This can be realized in an expanding universe in interactions involving very massive particles, in a first order phase transition, or due to
thermal domain walls around black holes as proposed in the present work.

Baryon non-conserving processes may exist at the grand unification energy scale [22] predicted by Grand Unified Theories (GUTs), supersymmetric models and even at the electroweak energy scale [23] predicted by the standard model. GUTs, as they unify electroweak with strong nuclear forces, treat quarks and leptons as members of the same families. It is natural, then, that in these theories neither baryon number $B$ nor lepton number $L$ to be conserved independently. It is rather $B - L$ that is conserved. At the Standard Model, now, the electroweak classical Lagrangian preserves baryon number. It is the quantum corrections of the vacuum state that violate it (sphaleron process). These predictions have not been observed yet.

C-symmetry violation is needed so that processes producing more baryons than antibaryons prevail over those producing more anti-baryons. CP-symmetry violation ensures that the generated left-handed baryons are not equal in number to the right-handed antibaryons (and the right-handed baryons to the left-handed anti-baryons).

C and CP-symmetry violation has been predicted theoretically and is observed. Weak interactions violate C and P. CP-violation is expressed in the quarks mixing matrix as a complex phase of $\Delta \theta$. This is also the case for the neutrino mixing matrix, but the values of the CP-violating angles are not known. The CP-violation predicted by the Standard Model is not adequate for the creation of the observed baryon asymmetry and thus some extension is required.

Thermal equilibrium would mean that both directions of some interactions that do not conserve the baryon number are equally active and so no net baryon asymmetry would be created. This is the case when the temperature of the universe is higher than the mass of some particle species and then the interactions that take place can both create and destroy particles of this species in equal rates. When the temperature falls below the value of this particle’s mass but some particles still live, then they can only decay. There is not enough energy for the creation of particles. This is a state of non-thermal equilibrium.

2.3 Models of Baryogenesis

Various interesting models of baryogenesis have been proposed during the last thirty years (see reviews [15]-[20]). An important piece of knowledge extracted from all this research is the realization that it has proved quite difficult to construct a simple model capable to generate the observed amount of baryon asymmetry.
Baryogenesis by heavy particles decay in Grand Unified Theories

Baryogenesis by heavy particle decays was the first baryogenesis model ever and it was proposed by Sakharov in [21] [24] (see also [24], [25], [26]). Due to heavy particle decays in an expanding cosmology in the presence of C and CP violation, baryon asymmetry is produced. Usually, this mechanism can be realized in grand unification models with the heavy particle being a gauge boson of grand unification (called X-boson), with a mass around $10^{15}\text{GeV}$, that is the energy scale of grand unification.

All three prerequisites of baryogenesis are satisfied. Baryon number violation is an intrinsic feature of grand unification since both quarks and leptons are treated as members of a common irreducible representation of the gauge group. This way there are interactions, mediated by gauge bosons, that transform quarks into leptons or antiquarks. These interactions do not conserve B. An X-boson can decay, for example, to $qq$ and $\bar{q}\bar{l}$ pairs. The channels that $X$ and its anti-particle $\bar{X}$ decay into, are shown in table (2.3.1). Baryon number B is not conserved, as $qq$ has $B_{qq} = \frac{2}{3}$, while for $\bar{q}\bar{l}$ is $B_{\bar{q}\bar{l}} = -\frac{1}{3}$. Note, though, that the difference between baryon and lepton numbers $B - L$ is conserved. This is true for several GUT models, as it is also for the electroweak theory.

<table>
<thead>
<tr>
<th>particle</th>
<th>decays into</th>
<th>$B$</th>
<th>branching ratio</th>
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<tr>
<td>$X$</td>
<td>$qq$</td>
<td>$\frac{2}{3}$</td>
<td>$r$</td>
</tr>
<tr>
<td>$X$</td>
<td>$\bar{q}\bar{l}$</td>
<td>$-\frac{1}{3}$</td>
<td>$1 - r$</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>$\bar{q}\bar{q}$</td>
<td>$-\frac{2}{3}$</td>
<td>$\bar{r}$</td>
</tr>
<tr>
<td>$X$</td>
<td>$ql$</td>
<td>$\frac{1}{3}$</td>
<td>$1 - \bar{r}$</td>
</tr>
</tbody>
</table>

Table 2.3.1 Decay of $X$ and $\bar{X}$.

The mean baryon number created by the decay of an $X$ is

$$B_X = \frac{2}{3}r - \frac{1}{3}(1 - r),$$ (6)

where $r$ is branching ratio, while that created by the decay of a $\bar{X}$

$$B_{\bar{X}} = -\frac{2}{3}\bar{r} + \frac{1}{3}(1 - \bar{r}).$$ (7)

The mean baryon number produced by the decay of an $X$ and a $\bar{X}$ is

$$\epsilon \equiv B_X + B_{\bar{X}} = r - \bar{r},$$ (8)
or, more generally

\[ \epsilon \equiv \sum_f B_f \frac{\Gamma(X \to f) - \Gamma(\bar{X} \to \bar{f})}{\Gamma_X}, \]  

(9)

where the sum is over all possible final states \( f \), \( B_f \) is the baryon number of this state \( f \), \( \Gamma_X \) is the total \( X \) decay width, while \( \Gamma(X \to f) \) is the partial decay width of the specific decay. If there is no C and CP violation, no net baryon number is produced, because \( r = \bar{r} \). We know, though, that C and CP are violated even at the low energies - compared to those of the grand unification scale - of the standard model. We assume that it is so at the GUT scale also. Moreover, the violation could be even maximal, in the sense that theory does not forbid it. \( \epsilon \), yet, can be as low as \( \sim 10^{-8} \) [59], for the production of the observed baryon excess.

Regarding the non-equilibrium, now, one can say that the decay of the very heavy X-bosons is such a process when the universe temperature has fallen below the GUT energy scale. This is so because then X-bosons decay but the temperature is insufficient for them to be reproduced.

One problem of GUT baryogenesis is that the universe temperature after inflation is probably below the GUT scale. Even in this case though, X-bosons could be created at the end of inflation by gravity. Another problem is that a process creating baryon excess can also wipe out any pre-existing baryon number. So, a later process could destroy the baryon excess created by heavy particles decay.

### 2.3.2 Electroweak Baryogenesis

Electroweak baryogenesis is another scenario that attracted a lot of study [27]. It is natural to expect to generate baryon asymmetry at this energy scale, otherwise rapid sphaleron processes at 100 GeV will destroy any baryon asymmetry produced earlier. The standard model incorporates non-conservation of baryons (through the chiral anomaly) [23], as well as deviations from thermal equilibrium if the phase transition is of the first order. Unfortunately, the recently discovered heavy Higgs boson turns the transition to a second order one. Another disadvantage of the standard model is its very small CP-violating phases [28]. The present work refers to the electroweak energy scale but solves both problems as it will be explained shortly. There are of course other possibilities too to create baryon number at the electroweak scale, like TeV scale gravity [29], [30].

The electroweak classical Lagrangian conserves baryon number. Quarks appear always in \( q\bar{q} \) combination and so a quark can be destroyed only in a collision with an antiquark.
Because of quantum corrections, though, the $B + L$ number of the vacuum state changes by

$$\Delta(B + L) = 2N_f N_{CS},$$  \hspace{1cm} (10)$$

where $N_f = 3$ is the number of families and $N_{CS}$ is the Chern-Simons number which measures the amount the phases of quantum fields have changed under the gauge transformation (in units of $2\pi$). $B - L$ is conserved, so the baryon number change is

$$\Delta B = \frac{1}{2} \Delta(B + L) N_f N_{CS}. \hspace{1cm} (11)$$

The result is a multiple of 3. This is the sphaleron process.

As said before, C and CP symmetries are broken in the standard model. Parity is maximally violated since only left handed particles interact with electroweak processes. CP is generally conserved in SM, except quark mixing, and so C is also not conserved. The quark mixing CP violation is small and it is found in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which is the quarks mixing matrix.

The third Zacharov criterion is the departure from thermal equilibrium. As the universe cools, because of expansion, below the electroweak energy scale ($\sim 100\text{GeV}$), a phase transition takes place from the symmetric phase of the higher temperatures to the phase with broken symmetry of lower temperatures. This transition is either of first or of second order. If it is of second order, then it is a smooth transition and the thermal equilibrium is not disturbed. If it is of first order, then bubbles of broken phase form in the unbroken phase background. The expansion of the bubbles is a considerable departure from thermal equilibrium. The type of phase transition depends on the Higgs mass. For high mass (above about $80\text{GeV}$), the transition is second order, while for low masses it is of first order. With a Higgs mass of $125\text{GeV}$, the transition is of second order and there is no electroweak baryogenesis. Our models, nevertheless, study electroweak baryogenesis by primordial black holes emission and do not require a first order phase transition.

### 2.3.3 Baryo-through-Leptogenesis

Another way to produce baryon asymmetry is baryo-through-leptogenesis [31]. At high energies, as large as $10^{10} \text{GeV}$, lepton asymmetry is produced from heavy Majorana neutrino decays, and subsequently, this lepton asymmetry leads to baryon asymmetry by the equilibrium electroweak processes which break $(B + L)$ but conserve $(B - L)$ symmetry [32].
The three SaKharov criteria are valid not only for baryogenesis, but for leptogenesis also. The first criterion, of course, refers to non-conserving lepton number processes. Heavy Majorana neutrinos have been suggested in the frame of the seesaw mechanism, in order to explain the light masses of the three known neutrinos. It is supposed that the light neutrinos mix with very heavy Majorana neutrinos in the mixing matrix. The decay of these heavy neutrinos does not conserve leptonic number $L$.

CP violation, now, may be easier for Majorana than Dirac fermions. The mass matrix of Dirac fermions contains one CP-odd phase. The 3x3 Majorana mass matrix involves three independent CP-odd phases. If we add three heavy neutrinos, then we'll have three phases in the light neutrinos sector and three more in the heavy neutrinos sector. Their values are unknown and so we can assume even large ones. The phases measured in neutrino oscillations are not directly linked to the phases in heavy neutrino decays. So, low energy measurements cannot reveal the CP violation in leptogenesis.

After the universe temperature has dropped below the heavy neutrinos energy scale, their decay is an off-equilibrium process, since they also cannot be created. This way all criteria are satisfied for leptogenesis.

Then, leptons excess over anti-leptons has to be transformed to baryons excess over anti-baryons. This happens at the electroweak scale by sphaleron processes. These processes take place in thermal equilibrium and conserve $C$ and $CP$ symmetry. Sphalerons do not conserve $B$ or $L$, but they conserve $(B - L)$. The lepton number previously created becomes both lepton and baryon number.

### 2.3.4 Baryogenesis by Primordial Black Holes

Another interesting class of models concerns baryogenesis through the evaporation of primordial black holes (PBHs) [35]. Primordial black holes can be very small black holes created at the first moments of the universe [40]. Hawking radiation that is emitted by a black hole can be considered as blackbody radiation. This does not produce any baryon asymmetry. Nevertheless, the particle propagation of the Hawking radiation [36] in the gravitational field of the BH distorts the thermal equilibrium and makes possible the creation of a net excess of particles over antiparticles, assuming baryon non-conserving heavy particle decays. Let us assume, for example, that a heavy meson $M$, emitted by a black hole, decays, just outside the event horizon, to a light baryon $B_L$ and a heavy
anti-baryon $\bar{B}_H$ or vice versa:

\begin{align}
M & \rightarrow B_L + \bar{B}_H , \\
M & \rightarrow B_H + \bar{B}_L .
\end{align}

There is a probability for a decay product to fall back into the black hole and it is higher for the heavy particle than for the light one. If now, because of C and CP asymmetry, the two decay channels are not of the same probability, then we end up with a baryon asymmetry generated by the black hole.

The first PBHs baryogenesis models were based on grand unified theories (GUT) [41]. GUT processes can truly produce baryon number, but this is subject to sphaleron washout [42]. The electroweak baryogenesis scenario proposed by Cohen, Kaplan and Nelson (CKN model) [43] addresses this problem by applying the sphaleron process to produce baryon number. Nagatani [44], in order to overcome the well-known problems of electroweak baryogenesis, proposed a scenario where baryogenesis takes place in a thermal domain wall surrounding small primordial black holes with a temperature higher than the electroweak critical temperature $T_W \simeq 100$ GeV. Although the idea in [44] is attractive, it did not receive much interest. One disadvantage is the assumption that the universe should pass through a black hole dominated era after inflation. This is not very natural, although not forbidden, in the context of 4-dimensional cosmology [70]. However, it becomes very possible in a universe with early high energy modifications, as e.g. in a brane world cosmology where the creation of PBHs is much easier due to the low 5-dim Planck scale. Another disadvantage is the final outcome that only for very large values of the CP-violating phase it is possible to produce the required baryon asymmetry. This is true for black holes masses around 100Kgs.

### 2.3.5 Other baryogenesis models

A different mechanism is the Affleck-Dine baryogenesis [33]. In supersymmetric models, scalar superpartners of baryons or leptons can acquire a large baryonic charge after inflation. Subsequent $B$-conserving decay of these fields transform baryon asymmetry into an asymmetry in the quark sector. This mechanism, contrary to all others, leads to a quite large value of the baryon-to-photon ratio of order one, and special effort is needed to generate the observed value. The mechanism proposed in the current work is also able to generate large baryon asymmetries which, however, can be controlled to have the correct value. Large amounts of baryon asymmetry can also be generated in the
so-called Spontaneous Baryogenesis [34], in which a spontaneously broken global $U(1)$ symmetry associated with the baryonic number is assumed to exist. Unfortunately, these models are associated with large isocurvature density perturbations at large scales which are forbidden by cosmic microwave background data.

There are finally some more exotic scenarios, for example mechanisms based on spatial separation of $B$ and $\bar{B}$ (these scenarios allow for baryonic charge conservation and globally baryo-symmetric universe) [37]-[39]. Successful baryogenesis could be realized even without the three Sakharov conditions [25], although these mechanisms are somewhat more complicated technically.
3 Randall - Sundrum II universe

3.1 Randall - Sundrum II model

Randall - Sundrum II (RS II) [45] is a model where our universe lies on a 4-dimensional brane embedded in a 5-d bulk. Extra dimensions models are built since Kaluza - Klein and string theories are a main source of such models. In many cases, the extra dimensions are considered compact. Free propagation of matter and energy at large distances in the extra dimensions would mean their non-conservation in the 4-d world we live in. We can have large extra dimensions if the Standard Model is confined on a 3-brane, that is with 3+1 dimensions, which lies in the higher dimensions space [29]. However, gravity cannot be confined, since it is the dynamics of spacetime. So, the extra dimensions can be non-compact, but yet small enough in order not to be in conflict with experimental and observational data (up to some mm).

The RS II model accommodates one extra dimension with even infinite volume and yet gravity effectively confined in 4-dimensions. Instead of being restricted by the size of the extra dimension, it is so by its curvature. Because of this curvature, the graviton is at a bound state in the extra dimension. Thus, although the fifth dimension is infinite, the graviton is confined in a very small region. The Newton law of gravitation and relativistic corrections are reproduced with adequate precision in 4-dimensions.

L. Randall and R. Sundrum considered our world on a brane with positive tension, embedded in a 5-dimensional anti-de Sitter bulk. They also considered a second brane at distance $\pi l$ from our world brane in the fifth dimension, where $l$ is the curvature of the extra dimension. They first used a similar technique in [46], where they presented the RS I model, in order to tackle the hierarchy problem. The usefulness of this second brane is to make the 5-dimension volume finite and so to ease calculations. At the end, the distance between the two branes can be set to be infinite. Then, the action of the system is

$$S = S_{gr} + S_{br} + S_{br}',$$

$$S_{gr} = \int d^4x \int dy \sqrt{-G} \{-\Lambda_5 + 2m_5^2 R\},$$

$$S_{br} = \int d^4x \sqrt{-g_{br}} \{\lambda_{br} + L_{br}\},$$

(13)

where $br$ is for our world’s brane, $br'$ is for the second brane and $gr$ is for gravity. $R$ is the 5-d Ricci scalar, $\Lambda_5$ and $\lambda_{br}$ are cosmological terms in bulk and branes. More specifically, $\Lambda_5$ is the five-dimensional cosmological constant and $\lambda_{br}$ is the tension of the brane. In
the following, $\lambda$ will be our world’s brane tension. $m_5$ is the 5-dimensional fundamental Planck mass.

The solution of the Einstein equations is
\[ ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \] (14)
where $x$ are the brane coordinates, while $y$ is the coordinate of the 5th dimension and takes values from 0 to $\pi l$. As one can see, the metric decreases exponentially from brane to bulk. The solution is valid for
\[ \lambda_{\text{br}} = -\lambda_{\text{br}}' = 24m_5^3k, \quad \Lambda_5 = -24m_5^3k^2. \] (15)
The meaning of this setting is that our world’s brane positive tension (and the second brane’s negative one) counterbalance the 5-dimensional cosmological constant.

It is very interesting then to see how the 4-d effective Planck mass $m_4$ is derived from the 5-d fundamental Planck mass $m_5$. We substitute $\eta_{\mu\nu}$ in eq. (14), with a four-dimensional metric $\bar{g}_{\mu\nu}(x)$ and we get the 4-d graviton zero-mode. We also get the four-dimensional Ricci scalar $\bar{R}$ from $\bar{g}_{\mu\nu}(x)$. Then, the action becomes
\[ S = \int d^4x \int_0^{\pi l} dy \, 2m_5^3r_e^{-2k|y|}\sqrt{|\bar{g}|} \bar{R}. \] (16)
We integrate with respect to $y$ and we have the 4-dimensional effective action. From this, we derive the 4-d effective Planck mass:
\[ m_4^2 = 2m_5^3\int_0^{\pi l} dy e^{-2k|y|} = \frac{m_5^3}{k}[1 - e^{-2k\pi}]. \] (17)

Adding now in eq. (14) a small fluctuation term $h_{\mu\nu}(x, y)$ to the metric tensor term $e^{-2k|y|}\eta_{\mu\nu}$, we can examine whether the linearized fluctuations equation is compatible with the 4-d observational and experimental data of gravity. A Kaluza-Klein (KK) reduction in the 4 dimensions is needed. In order to do so, we separate the 5th dimension variable from the other four:
\[ h(x, y) = \psi(y)e^{ipx}, \] (18)
where $p^2 = m^2$. Then, eq. (14) becomes the linearized equation for the small fluctuations:
\[ \left[ -\frac{m^2}{2} e^{2k|y|} - \frac{1}{2} \partial_y^2 - 2k\delta(y) + 2k^2 \right] \psi(y) = 0. \] (19)
With a change of variable $z = \text{sgn}(y) \left(e^{k|y|} - 1 \right)/k$, $\hat{\psi}(z) = \psi(y)e^{k|y|/2}$, $\hat{h}(x, z) = h(x, y)e^{k|y|/2}$ eq. (19) takes the form of a non-relativistic quantum mechanics equation:
\[ \left[ -\frac{1}{2} \partial_z^2 + V(z) \right] \hat{\psi}(z) = m^2 \hat{\psi}, \] (20)
where

\[ V(z) = \frac{15k^2}{8(k|z| + 1)^2} - \frac{3k}{2} \delta(z). \]  

(21)

Because of \( \delta(z) \), there is a single normalizable bound state mode, which can be identified as the graviton:

\[ \hat{\psi}_0(z) = k^{-1}(k|z| + 1)^{-3/2}. \]  

(22)

Then, there is a continuum of higher KK modes. There is no gap, as in other models with extra dimensions, since \( V(z) \to 0 \) as \( |z| \to \infty \). The amplitudes of the continuum modes are, also, suppressed near the brane, because \( V \to \infty \) when \( z \to 0 \). The masses of the continuum KK states have all the values from 0 to infinity.

Using both the zero-mode and the KK modes, we can check whether the non-relativistic Newtonian potential is reproduced on the brane. The static potential generated by the exchange of the zero and KK-modes propagators between two masses \( m_1 \) and \( m_2 \) on the brane \( (z = 0) \) is

\[ V(r) \sim G_N \frac{m_1m_2}{r} + \int_0^\infty \frac{dm}{k} G_N \frac{m_1m_2e^{-mr}}{k}. \]  

(23)

The first term is the Newtonian gravitational potential, generated by the exchange of the zero-mode propagator, which is the massless graviton of our world. The second term is the integral over the higher KK modes. Because of the exponential Yukawa suppression for \( m > 1/r \), it becomes

\[ V(r) = G_N \frac{m_1m_2}{r} \left(1 + \frac{1}{r^2k^2}\right). \]  

(24)

The KK corrections term is extremely suppressed because \( k \) is of the order of the 5-d Planck mass \( m_5 \) and for \( r \) at least the size of the experimental limit. So, Newtonian gravity is restored on our brane. Moreover, the propagators are relativistic and they generate the correct relativistic correction to the non-relativistic approximation. The corrections derived from the KK modes remain negligible.

### 3.2 Cosmology in RS II universe

The cosmology derived from the equations of the RS II model, has been studied in several papers [47]. We take the energy-momentum tensor to be confined on the brane and we consider it as a perfect fluid. We also assume that the four-dimensional metric on the brane is Friedmann-Robertson-Walker. Then, the energy-momentum conservation equation on the brane is the same as in standard cosmology:

\[ \dot{\rho} + 3H(\rho + p) = 0, \]  

(25)
while the second Friedmann equation is modified:

\[ H^2 = \frac{8\pi}{3m_4^2} \left( \rho + \frac{\rho^2}{2\lambda} + \rho_{KK} \right) + \frac{\Lambda_4}{3} - \frac{k}{a^2}. \tag{26} \]

In the above equations, \( \rho \) is the fluid density and \( p \) its pressure, \( H \) the Hubble constant, \( \alpha \) the scale factor and \( k = -1, 0, 1 \) for open, flat or closed brane respectively. The dot over \( \rho \) is derivative with respect to the cosmic time \( t \). \( \rho_{KK} \) is the effective density on the brane of the Kaluza-Klein modes on the brane and we can accept it as negligible. Since our universe expands with only a small value of acceleration, we set the four-dimensional cosmological constant \( \Lambda_4 = 0 \). Finally, \( k = 0 \), because we live in a flat universe.

After these simplifications, we see that only two terms survive in the second part of the Friedmann equation (26):

\[ H^2 = \frac{8\pi}{3m_4^2} \left( \rho + \frac{\rho^2}{2\lambda} \right). \tag{27} \]

The \( \rho \) term is the same to the one present in standard cosmology too, while we have an additional \( \rho^2 \) term. For late times, when the density is low, we can neglect the \( \rho^2 \) term and then, the equation becomes the same as in standard cosmology. At early times, though, when the energy density is much higher, \( \rho^2 \) term is the leading term and this is a deviation from standard cosmology. This is the ‘high-energy’ regime. The solutions are different, as we will see in a little. Because of this, the evolution of primordial black holes is also different, which is what interests us most since we develop a model for baryogenesis by means of primordial black holes.

The relation between the brane tension \( \lambda \), the fundamental 5-dimensional Planck mass \( m_5 \) and the 4-dimensional effective Planck mass \( m_4 \) is given by

\[ \lambda = \frac{3M_6^5}{4\pi M_4^2}. \tag{28} \]

The AdS curvature \( l \), in relation with \( \Lambda_5 \) and \( m_5 \), is

\[ \Lambda_5 = -\frac{3}{4\pi} \frac{m_5^3}{l^2}. \tag{29} \]

The four-dimensional cosmological constant is

\[ \Lambda_4 = 3 \left( \frac{m_5^6}{m_4^4} - \frac{1}{l^2} \right) \tag{30} \]

and since we set it equal to zero, it is

\[ \lambda^{-1/4} = \left( \frac{4\pi}{3} \right)^{1/4} \left( \frac{l}{l_4} \right)^{1/2} l_4, \tag{31} \]

18
where \( l_4 = m_4^{-1} \) is the 4-dimensional Planck length.

The solutions for the energy density and the scale factor are:

\[
\rho = \frac{3m_4^2}{32\pi} \frac{1}{t(t + t_c)},
\]

(32)

\[
a = a_0 \left[ \frac{t(t + t_c)}{t_0(t_0 + t_c)} \right]^{1/4},
\]

(33)

where \( t_0 \) is any non-zero time, while \( t_c \) is the ‘transition time’ from the high-energy regime to the low-energy regime. It is given by

\[
t_c \equiv \frac{l_4}{2}.
\]

(34)

In the high-energy regime, that is \( t \ll t_c \Leftrightarrow \rho \gg \lambda \), the energy density and the scale factor become

\[
a = a_0 \left( \frac{t}{t_0} \right)^{1/4},
\]

(35)

\[
\rho = \frac{3m_4^2}{32\pi t_c t}.
\]

(36)

For \( t \gg t_c \Leftrightarrow \rho \ll \lambda \), we have the low-energy regime and standard cosmology is restored:

\[
a = a_0 \left( \frac{t}{t_0^{1/2} t_c^{1/2}} \right)^{1/2},
\]

(37)

\[
\rho = \frac{3m_4^2}{32\pi t^2}.
\]

(38)

The temperature of the radiation now is given by the same formula in high-energy regime, as in standard cosmology:

\[
\rho = \frac{\pi^2}{30} g T^4,
\]

(39)

where \( g \) is the number of relativistic particle species. Because of the modified energy density though, the relation of temperature to time is altered:

\[
\frac{T}{T_4} = \left( \frac{45}{8\pi^2} \right)^{1/4} g^{-1/4} l^{-1/4} \left( \frac{t}{l_4} \right)^{-1/4},
\]

(40)

where \( T_4 \) is the four-dimensional Planck temperature.
3.3 Primordial black holes in RS II cosmology

We are interested in very small primordial black holes that generate the baryon number of the universe through their evaporation. Neglecting charge and rotation and with the preposition that the size of the extra dimensions is much larger than the size of the black hole event horizon, then the Schwarzschild solution of the Einstein equations in n-dimensions \((n \geq 4)\), is \[ds^2_n = -f(r) \, dt^2 + f^{-1}(r) \, dr^2 + r^2 d\Omega^2_{n-2},\] where \[f(r) = 1 - \left(\frac{r_{BH}}{r}\right)^{n-3},\] \(r_{BH}\) is the Schwarzschild radius and \(d\Omega_{n-2}\) is the volume element of a \((n-2)\)-sphere. Thus, for a small black hole in the RS II universe and neglecting charge and rotation, the 5-dimensional Schwarzschild solution is

\[ds^2_5 = -(1 - \frac{r^2_{BH}}{r^2}) \, dt^2 + (1 - \frac{r^2_{BH}}{r^2})^{-1} \, dr^2 + r^2 d\Omega^2_3.\] \hspace{1cm} (43)

On the brane and near the event horizon, the effective 4-dimensional metric becomes

\[ds^2_4 = -(1 - \frac{r^2_{BH}}{r}) \, dt^2 + (1 - \frac{r^2_{BH}}{r})^{-1} \, dr^2 + r^2 d\Omega^2_2.\] \hspace{1cm} (44)

This is not the same as the known Schwarzschild solution in four dimensions (derived from eq. 41 for \(n = 4\)):

\[ds^2_4 = -(1 - \frac{r_{BH}}{r}) \, dt^2 + (1 - \frac{r_{BH}}{r})^{-1} \, dr^2 + r^2 d\Omega^2_2.\] \hspace{1cm} (45)

The \(r \to \infty\) limit, of course, is the same: Minkowski metric.

Now, the radius, area and temperature of a black hole are [49]:

\[r_{BH} = \sqrt{\frac{8}{3\pi} \frac{m_{BH}^{1/2}}{m_5^{3/2}}} = \sqrt{\frac{8}{3\pi} \left(\frac{l}{l_4}\right)^{1/2} \left(\frac{m_{BH}}{m_4}\right)^{1/2} l_4},\] \hspace{1cm} (46)

\[A_5 = 2\pi^2 r_{BH}^3,\] \hspace{1cm} (47)

\[T_{BH} = \frac{1}{2\pi r_{BH}},\] \hspace{1cm} (48)

and are valid for \(r_{BH} \ll l\). This temperature formula differs from that of the 4-dimensional black hole:

\[T_{BH}(4D) = \frac{m_4^2}{8\pi m_{BH}}.\] \hspace{1cm} (49)
The comparison shows that black holes in the high energy regime of RS II model are colder than black holes of the same mass in the 4-dimensional universe.

Black holes emit Hawking radiation which consists of all kinds of particles with rest mass lower than black hole temperature. The generated baryon number is the difference of the emitted baryons and anti-baryons. It is of great importance, thus, to study evaporation - and then accretion also - during black hole’s lifetime [49].

The number of particles of a certain species \(i\), emitted in \(n\)–dimensional spacetime by a black hole of temperature \(T_{BH}\), in a time interval \(dt\) and with momentum from \(k\) to \(k + dk\) is

\[
dN_i = \sigma_i(k) \frac{dt}{e^{\omega/T_{BH}} + 1} \frac{d^{n-1}k}{(2\pi)^{n-1}}, \tag{50}
\]

were \(\omega^2 = k^2 + m^2\), with \(m\) being the mass of the particle. Plus and minus are for fermions and bosons respectively. \(\sigma_i\) are the cross-sections for absorption or emission. The rate of energy evaporated is then

\[
\frac{dm_{BH}}{dt} = -\sum_i \int \sigma_i(k) \frac{\omega}{e^{\omega/T_{BH}} + 1} \frac{d^{n-1}k}{(2\pi)^{n-1}}. \tag{51}
\]

For high frequencies (\(\omega \gg T_{BH}\)), all cross-sections become almost the same:

\[
\sigma \approx \frac{\Lambda_{\text{eff},n}}{4} = \frac{\Omega_{n-2} r_{\text{eff},n}^{n-2}}{4}, \tag{52}
\]

with \(\Omega_{n-2}\) the volume of a \((n-2)\)-sphere and \(r_{\text{eff},n}\) an effective radius for black-body emission [48]

\[
r_{\text{eff},n} = \left(\frac{n-1}{2}\right)^{1/(n-3)} \left(\frac{n-1}{n-3}\right)^{1/2} r_{BH}. \tag{53}
\]

Integrating over momenta and summing over all emitted species eq. (51) gives Stefan’s law:

\[
\frac{dm_{BH}}{dt} \approx -g_n \tilde{\sigma}_n A_{\text{eff},n} T^n, \tag{54}
\]

were \(g_n\) is composed of bosonic and fermionic degrees of freedom:

\[
g_n = g_{n,\text{bos}} + \frac{2^{n-1} - 1}{2^{n-1}} g_{n,\text{ferm}} \tag{55}
\]

and \(\tilde{\sigma}_n\) denotes the \(n\)-dimensional Stefan–Boltzmann constant per degree of freedom:

\[
\tilde{\sigma}_n = \frac{\Omega_{n-2}}{4 (2\pi)^{n-1}} \Gamma(n) \zeta(n). \tag{56}
\]

\(\zeta(D)\) is the Riemann zeta function.
Separating emission to bulk from emission to brane, we have:

\[
\frac{dm_{BH}}{dt} \approx -g_{brane}\tilde{\sigma}_4 A_{eff,4} T^4 - g_{bulk}\tilde{\sigma}_5 A_{eff,5} T^5,
\]

where \(A_{eff,4} = 4\pi r_{eff,4}^2\). For \(g_{bulk}\) we count only the 5 polarization states of the graviton and not Kaluza Klein modes, which are effective modes in 4-dimensions, not in 5-dimensions. We remind that all other particles are confined on the brane. For our model of baryogenesis we consider primordial black holes small enough, that are hot enough, to produce all standard model particle species.

We integrate then eq. (57) and we get the lifetime of a black hole with initial mass \(m_{BH}\):

\[
t_{evap}(m_{BH}) \approx \tilde{g}^{-1} l_4 \left(\frac{m_{BH}}{m_4}\right)^2 t_4,
\]

were

\[
\tilde{g} = \frac{1}{160} g_{brane} + \frac{9\zeta(5)}{32\pi^4} g_{bulk}.
\]

\(g_{bulk} = 5\) (the graviton polarization states) while \(g_{brane} = g_{eSM} = 106.75\) for the small black holes we are interested in and so emission to brane is of minor significance.

The lifetime of a black hole in RS II is much longer than that of a black hole of the same initial mass in standard 4-dimensional cosmology:

\[
t_{evap}(m_{BH}, 4d) \approx 1.2 \times 10^4 \tilde{g}^{-1} \left(\frac{m_{BH}}{m_4}\right)^3 t_4,
\]

\[
\frac{t_{evap}(m_{BH}, 5d)}{t_{evap}(m_{BH}, 4d)} \sim \left(\frac{l}{r_{BH}(5d)}\right)^2.
\]

### 3.3.1 Accretion

Accretion plays a crucial role in our baryogenesis model. It can lead the universe to become black hole dominated, even if black hole density is relatively low at the time of their creation. The lifetime of the black holes is also increased. Both contribute to enhanced baryon number generation.

Accretion by primordial black holes in standard 4-dimensional cosmology is considered inefficient [50], [51]. The case is not so in RS II because of the existence of the high-energy regime. This is exactly what we are interested in because our model needs very small and hot primordial black holes, born in the high-energy regime. In later times there is
no deviation from standard cosmology. Accretion by primordial black holes in RS II is discussed in [49], [52].

Accretion rate, that is power absorbed by a black hole, is proportional to the black hole surface area and the radiation density surrounding the black hole. The trajectories of particles now that are absorbed, are bent to the black hole before crossing the event horizon. Therefore, an effective radius $r_{\text{eff}} > r_{BH}$ is used, instead of event horizon radius $r_{BH}$, as BH surface area. For the 5-dimensional primordial black holes in RS II, it was estimated in [48] that $r_{\text{eff,5}} = 2r_{BH}$. So, it is

$$\frac{dm_{BH,\text{acc}}}{dt} = F \pi r_{\text{eff,5}}^2 \rho_{\text{rad}}, \quad (62)$$

where $F$ is the coefficient that denotes accretion efficiency and it is $F \leq 1$. There is an uncertainty about the value of $F$, but if the radiation particles mean free path is greater than the event horizon - and this is the case for our model since the PBHs in it are tiny - then $F$ can be close to 1.

We remind that the energy density in the high energy regime is

$$\rho = \frac{3m_4^2}{32\pi t_c t}. \quad (63)$$

Substituting this and also $r_{\text{eff,5}}$ in equation (62) the latter becomes

$$\frac{dm_{BH}}{dt} = \frac{2F}{\pi} \frac{m_{BH}}{t}. \quad (64)$$

Integrating, we have

$$m_{BH}(t) = m_i \left( \frac{t}{t_i} \right)^{2F/\pi}. \quad (65)$$

where $m_i$ is the initial black hole mass. We see that the mass increase of the black hole can be considerable, especially for $F$ close to 1.

After the high-energy epoch ends at the transition time $t_c$, the density of radiation is given by equation(38). It is now inversely proportional to $t^2$, not $t$. It is diluted more rapidly and so accretion becomes less effective. The solution of eq. (62) now becomes

$$m_{BH}(t) = m_{BH}(t_c) \exp \left[ \frac{2F}{\pi} \left( 1 - \frac{t_c}{t} \right) \right], \quad (66)$$

where $m_{BH}(t_c)$ is black hole's mass at the transition time. We see that accretion continues but only for a small time duration after $t_c$ and giving only a small mass increase.
Up until now we have not combined accretion with evaporation. Taking both phenomena into account, the equation that governs black hole mass evolution becomes

\[
\frac{dm_{BH}}{dt} = \left( \frac{dm_{BH}}{dt} \right)_{acc} + \left( \frac{dm_{BH}}{dt} \right)_{evap}
\]

\[
\frac{2F}{\pi} \frac{m_{BH}}{t} - \frac{\tilde{g} m_{BH}^3}{2\dot{m}_{BH}}.
\]

In the high-energy regime the solution is

\[
m_{BH}(t) = \left\{ \left( \frac{t}{t_i} \right)^q - \frac{\tilde{g}}{4\sqrt{f}} \frac{1}{1 - \frac{m_{BH}}{m_i}^{3/2}} \left[ \left( \frac{t}{t_i} \right) - \left( \frac{t}{t_i} \right)^q \right] \right\}^{1/2} m_i,
\]

where \( q = \frac{4F}{\pi} \). For a wide range of parameters accretion is dominant during the high-energy epoch. For the purpose of our baryogenesis model, we are not going to use the above relation, since it was derived under the assumption of radiation domination. In our model universe turns black-hole dominated, until of course their complete evaporation.
4 Baryogenesis by primordial black holes in Randall-Sundrum II universe

The present chapter is based on [1]. We first correct a dubious constraint that was used in [44] and find that the allowed parameter space for baryogenesis is improved. We then study this baryogenesis mechanism in the context of Randall-Sundrum (RS) brane cosmology [45] and explain how it is possible to very easily get efficient generation of baryon asymmetry even for very small CP-violating angles. The allowed by this mechanism BH mass range includes masses around the higher dimensional Planck mass. The latter is important since this mass spectrum is energetically favorable to be generated from high energy interactions in the very early braneworld cosmic history. Furthermore, the black hole domination era can now be naturally realized due to the accretion in the high energy regime.

Let us explain in more detail the proposed scenario. The existence of extra dimensions [53] is considered possible and a lot of research has been carried out towards higher dimensional cosmological models. In this case not only the cosmic geometry, but also the properties of black holes in theories with large or infinite extra dimensions are different, since now the fundamental Planck mass lies much lower. The proposed scenario of electroweak baryogenesis concerns the baryon asymmetry generation at the domain wall around annihilating PBHs in a universe with extra dimensions and, in particular, RS-II cosmology [45]. There are various mechanisms for generating these PBHs. After their formation, a part of the universe consists of PBHs and the rest consists of radiation at temperatures lower than the electroweak scale. Soon after their formation, PBHs start to accrete and evaporate. Depending on the accretion efficiency the two phenomena can dominate each other. The emitted black hole Hawking radiation thermalises the surrounding region at temperatures above the electroweak scale. This results to the creation of a domain wall that connects the two different vacua. As the Hawking particles pass this domain wall, they experience a CP violation leaving a net baryon asymmetry in the outgoing emitted radiation. At the end of PBHs’ complete evaporation, the Universe is reheated from this Hawking flux at temperatures above the nucleosynthesis scale. The produced baryogenesis is greatly enhanced due to the extended lifetime of the PBHs. The prolonged lifetime is caused by the accretion factor which holds at the high energy cosmic period. In addition, the significant black hole accretion that takes place, allows a black hole dominated cosmic era which helps the mechanism.

Electroweak baryogenesis takes place at the domain wall via the standard sphaleron...
process [43]. Note that the existence of the symmetric region surrounding the black hole washes out any baryon number created in a prior epoch. The proposed scenario satisfies Sakharov’s three criteria for baryogenesis [54], [44]. First, the sphaleron process that takes place at the domain wall is a baryon number violating mechanism. Second, although the Standard Model is a chiral theory that incorporates C-asymmetry, this is not large enough. Thus, we assume a two-Higgs doublets extension of the Standard Model [55], [56] as the background field theory, because it provides large CP violating phases on the Higgs sector. Finally, the outgoing radiation of the black hole is a non-equilibrium process. A main advantage of this scenario, compared to the CKN electroweak baryogenesis, is the type of the phase transition needed. At the CKN model a first order transition is required. In the present work the domain wall is created by the thermal radiation of the black hole, and so, the phase transition can be of second order [44]. The most important result of this study is the achievement of the observed value of $b/s \approx 6 \times 10^{-10}$ for very small CP violating angles.

4.1 Baryogenesis in the standard 4-dim FRW universe

As mentioned before, the possibility of electroweak baryogenesis by small primordial black holes in the standard 4-dimensional FRW universe was shown in [44]. In order to calculate the baryon-to-entropy ratio $b/s$, the author used for the black holes density the Einstein equation for a flat universe with matter domination:

$$\rho_{BH} = \frac{1}{6\pi} \frac{m_{Pl}^2}{t^2}. \quad (70)$$

Although the black hole dominated era can be described by $\rho_{BH} \propto a^{-3}$, it is not correct to fix the unknown integration constant and thus, this equation is correct up to an unknown prefactor. Indeed, we can not normalise to the present cosmic density since black holes completely evaporate. In addition, it is not possible to determine the prefactor using an initial black hole density at some initial time, since both these quantities are unknown and model dependent. In the discussed scenario the initial cosmic density $\rho_{BH}$ at formation is a free parameter since it depends on the details of the black holes generation (inflation or other mechanism). The correction of this mistake means that there is one less constraint for the black hole mass and the scenario becomes more attractive.

Let us now correctly estimate the amount of the produced baryon-to-entropy ratio.
The total baryon number created in the lifetime of a black hole is

\[ B = \frac{15}{4\pi^3 g_*} N \kappa \alpha_W^5 \epsilon \Delta \varphi_{CP} \frac{m^2_{pl}}{T_{BH} T_W}, \]  

(71)

where \( g_* \simeq 100 \) is the number of degrees of freedom that a BH can decay into at the electroweak temperature, \( N \simeq O(1) \) is a model dependent constant which is determined by the type of spontaneous electroweak baryogenesis scenario and the fermion content, \( \kappa \simeq O(30) \) is a numerical constant expressing the strength of the sphaleron process [57], \( \alpha_W = 1/30 \) [58], \( \epsilon \simeq 1/100 \), \( T_W = 100 \) GeV is the electroweak scale, and \( \Delta \varphi_{CP} \) is the CP violating angle. The total baryon number density created from all black holes is given by \( b = B n_{BH} \), where \( n_{BH} = \frac{\rho_{BH}}{m_{BH}} \) is the number density of the black holes assuming a monochromatic spectrum of black holes. The universe after the creation of PBHs is black hole dominated with density \( \rho_{BH} \). Soon after black holes almost instantaneous evaporation, the universe is reheated, its density has the form of radiation and is equal to \( \rho_{rad} (t_{reh}) \). Thus,

\[ \rho_{BH}(t_{reh}) \simeq \rho_{rad}(t_{reh}) = \frac{\pi^2}{30} g_{reh} T_{reh}^4. \]  

(72)

The fact that \( \rho_{BH} \) is a free parameter allows a freedom on the choice of \( T_{reh} \). However, \( T_{reh} \) has to be below \( T_W \) in order the baryogenesis scenario under discussion to be viable. Note that even if the scenario was working giving finally a baryon asymmetry at reheating temperature larger than \( T_W \), this asymmetry would be washed out later when the universe will experience the electroweak transition. In addition, \( T_{reh} \) has to be also larger than the nucleosynthesis temperature. Finally, estimating the cosmic entropy density as \( s = \frac{2\pi^2}{45} g_{reh} T_{reh}^3 \) [59], we can calculate the total baryon-to-entropy ratio asymmetry.

Choosing \( T_{reh} = 90 \) GeV it is possible to calculate the total baryon-to-entropy ratio from all black holes as

\[ \frac{b}{s} = 2.4 \times 10^{-10} \Delta \varphi_{CP}. \]  

(73)

It is obvious that the required for nucleosynthesis amount of baryon asymmetry is achieved only for \( \Delta \varphi_{CP} = \pi \). Therefore, this mechanism can hardly provide sufficient baryon asymmetry. For smaller values of \( T_{reh} \) the baryon asymmetry is further reduced. Eq. (73) surprisingly, does not depend on the black hole mass. Although the black hole mass does not determine the baryon asymmetry, it is constrained from two requirements regarding the existence of thermal stationary domain wall and the black hole lifetime in comparison with the domain wall time scale. These two constraints remain the same as in [44] and give the following range for the initial BH mass

\[ 4.3 \times 10^{28} \text{GeV} < m_{BH} < 1.1 \times 10^{32} \text{GeV}. \]  

(74)
4.2 Baryogenesis in the braneworld and black hole mass constraints

In the framework of a RS-II braneworld embedded in a AdS bulk, primordial black holes are produced after the end of inflation or at the very beginning of a non-inflationary flat model. Our scenario does not depend on the details regarding the origin of the PBHs, thus, this work does not study this issue. It is well known that the properties of the brane black holes are modified compared to those in a standard cosmology \cite{49}, \cite{52}. They are colder and live longer. More important, the accretion of material from the neighborhood of the black hole can be stronger than evaporation during a high-energy regime. This can not occur in the four-dimensional case. The presence of strong accretion has two advantages. First, it can lead to a black hole dominated universe, and second, it leads to an extension of the black holes lifetime.

Brane black holes involved in the proposed mechanism are small enough to ensure that the Hawking temperature $T_{BH}$ is much higher than the electroweak critical temperature $T_{W}$, and so, all kinds of Standard Model (SM) particles emitted on the brane, are in the symmetric phase. There is also emission towards the bulk, although much less, where only gravitons are assumed to radiate. Being interested in baryogenesis, we have to deal with the emission on the brane. The emission on the brane causes the thermalization of the surrounding region which contains radiation at low temperature. Local thermalization applies in a region where particles have a mean free path (MFP) smaller than the size of this region. Thus, a local temperature $T(r)$ can be defined, and the mean free path of a particle $f$ is given by $\lambda_f(T) = \frac{\beta_f}{T}$, where $\beta_f$ is a constant depending on the particle species only. The quarks and the gluons have a strong interaction and they have the shortest MFP with $\beta_s \simeq 10$.

For a black hole with temperature $T_{BH}$ there is always a closely neighborhood surrounding the horizon, which is not thermalized, with depth $\lambda_s$. Moreover, in our case, the 5-dim Schwarzschild radius is much smaller than $\lambda_s$, i.e. $r_{BH} = \frac{1}{2\pi} \frac{1}{T_{BH}} \ll \lambda_s$. This expression for the black hole radius $r_{BH}$ in terms of the temperature $T_{BH}$ is the 5-dim one. The radius and the area of the black hole that will be used in the estimations are given.
by [60]

\[ r_{BH} = \sqrt{\frac{8}{3\pi} \frac{m_{BH}^{1/2}}{m_5^{3/2}}} \]

\[ A_{BH} = 2\pi^2 r_{BH}^3 \]

(75)

which are valid provided that \( r_{BH} << l \). The quantity \( m_5 \) is the 5-dim fundamental Planck mass and \( l \) is the AdS radius. Due to the non-thermalization of the close neighborhood, the radiated particles propagate freely therein. This is why most particles radiated do not return to the black hole, and so, the flux of the Hawking radiation obeys the Stefan-Boltzmann law without corrections. The outer region that is thermalized [44] has as boundary the sphere with radius \( r_o = r_{BH} + \lambda_s \simeq \lambda_s \), which is a function of the local temperature \( T_o = \frac{\beta_s}{r_o} \). The radius \( r_o \) is the minimal thermalized radius and the temperature \( T_o \) is the boundary temperature.

Now, we consider the transfer equation of the energy in the thermalized region in order to determine the temperature distribution \( T(r) \), assuming the diffusion approximation of photon transfer at the deep light-depth region [61]. The energy diffusion current in Local Temperature Equilibrium (LTE) is \( J_\mu = -\frac{\beta}{\pi T(r)} \partial_\mu \rho \). In our case, the radiation density is \( \rho = \frac{\pi^2}{30} g_{*SM} T^4(r) \), where \( g_{*SM} \equiv \sum_f g_{sf} = 106.75 \) is the number of degrees of freedom of all the massless particles on the brane. This is approximately equal to the massless degrees of freedom in the SM. Also \( \beta/T \) is the effective MFP for all the particles and all the interactions on the brane with \( \beta \simeq 100 \). The transfer equation is \( \frac{\partial}{\partial t} \rho = -\nabla_\mu J^\mu \). It is possible to find a stationary spherical-symmetric solution [61] which is

\[ T(r) = \left[ T_{br}^3 + (T_o^3 - T_{br}^3) \frac{r_o}{r} \right]^{1/3}. \]

(77)

For this solution to be valid, we assume that the number of degrees of freedom of the massless particles \( g_{*SM} \) is approximately constant, which is valid for the region that interests us in the domain wall. \( T_{br} = T(r \to \infty) \) is the background brane temperature. This temperature \( T_{br} \) is brane model dependent and is related to the specific mechanism that created the primordial black holes which dominated the universe. The value of \( T_{br} \) can be as large as a temperature somewhat lower than \( T_W \) where sphaleron rate is suppressed, and as low as zero. A very small \( T_{br} \) can be realized in a particular PBHs production model or in a scenario where the continuous accretion of PBHs made the background almost empty.

The outgoing diffusion flux is

\[ F = 4\pi r^2 J(r) \simeq \frac{8\pi^3}{135} \beta_s \beta_g g_{*SM} \left[ 1 - (T_{br}/T_o)^3 \right] T_o^2. \]

(78)
This flux must be equal to the flux of the Hawking radiation

\[ \mathcal{F}_{BH} = 4\pi r_{BH}^2 \times \frac{\pi^2}{120} g_s T_{BH}^4 + 2\pi^2 r_{BH}^3 \times \zeta g_{bulk} T_{BH}^5, \]  

where \( \zeta \) is a constant. The radiation towards the bulk can be neglected [49, 62] because the five-dimensional flux is negligible due to the small value of \( g_{bulk} \). The relation \( \mathcal{F}_{BH} = \mathcal{F} \) gives the temperature \( T_o \) of the minimum thermalized sphere with radius \( r_o \)

\[ r_o = \frac{16\pi}{3} \left( \beta s \beta s \right)^{1/2} \left[ 1 - \left( \frac{T_{br}}{T_o} \right)^3 \right]^{1/2} \frac{1}{T_{BH}}, \]  

and

\[ T_o = \frac{16\pi}{3\beta s} \left[ 1 - \left( \frac{T_{br}}{T_o} \right)^3 \right]^{-1/2} T_{BH}. \]

Assuming that \( T_{br} \ll T_o \) the spherical thermal distribution surrounding the black hole is

\[ T(r) = \left( T_{br}^3 + \frac{9}{256\pi^2} \frac{T_{BH}^2}{r} \right)^{1/3} \]  

for \( r > r_o \).

Now, we will discuss the formation of a domain wall around the black hole. A two-Higgs doublet model will be assumed, since it is a quite general model that can include the supersymmetric Higgs sector. The symmetry is restored at the close neighborhood of the black hole, because of its high temperature. At a greater distance, the temperature falls below the electroweak scale and the vacuum is broken. Therefore, an electroweak domain wall forms around the black hole, starting at radius \( r_{DW} \). The mechanism presented does not depend on the phase transition order and so it does not have to be first order. A second order transition, which looks favorable, was adopted. The vacuum expectation value (vev) of the Higgs doublets depends on the distance \( r \) from the black hole center.

The two-Higgs scalar potential can be written as follows [63]

\[ V_{Higgs} = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 ((\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2) + V_D, \]  

where \( \lambda_i \) are real numbers and \( \Phi_1^\dagger = (\phi_1 + i\phi_2, \phi_3 + i\phi_4), \Phi_2^\dagger = (\phi_5 + i\phi_6, \phi_7 + i\phi_8) \).

Here, both weak isospin doublets have weak hypercharge \( Y_{weak} = +1 \). We follow the notation of [63] in which both Higgs doublet fields have the same hypercharge. The above potential, with the exception of \( V_D \) which we discuss in the following, is the most general potential satisfying the following discrete symmetries:

\[ \Phi_2 \rightarrow -\Phi_2, \; \Phi_1 \rightarrow \Phi_1, \; d_R^i \rightarrow -d_R^i, \; u_R^i \rightarrow u_R^i, \]  

(84)
where $u_R^i$ and $d_R^i$ represent the right-handed weak eigenstates with charges $\frac{2}{3}$ and $-\frac{1}{3}$ respectively. All other fields involved remain intact under the above discrete symmetries. These symmetries force all the quarks of a given charge to interact with only one doublet. Thus, Higgs mediated flavor changing neutral currents are absent. If the discrete symmetry is broken during a cosmological phase transition, it produces stable domain walls via the Kibble mechanism [64]. This problem can be solved by adding terms which break this symmetry, providing at the same time the required explicit CP violation for baryogenesis. The most general form of that part of the potential which breaks this discrete symmetry is

$$V_D = -\mu_3^2 \Phi_1 \Phi_2 + \lambda_6 (\Phi_1^* \Phi_1) (\Phi_2^* \Phi_2) + \lambda_7 (\Phi_2^* \Phi_2) (\Phi_1^* \Phi_2) + h.c.$$

This is commonly named as $D-$breaking part. The parameters $\mu_3$, $\lambda_6$ and $\lambda_7$ are in general complex numbers

$$\mu_3^2 = m_3^2 e^{i \theta_3}, \quad \lambda_6 = l_6 e^{i \theta_6}, \quad \lambda_7 = l_7 e^{i \theta_7},$$

providing explicit CP violation at tree level.

In order to study the structure of the vacua, we can perform an $SU(2)$ rotation that sets the vev’s of the fields $\phi_{1,2,4}$ equal to zero. Solving the system $\partial V_{\text{Higgs}} / \partial \phi_i = 0$ implies several different stationary points. One of them is the usual asymmetric minimum that respects the $U(1)$ of electromagnetism

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ u \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v e^{i \varphi_t} \end{pmatrix}. \quad (87)$$

In Eq. (186) $u, v, \varphi_t$ are real numbers. The phase $\varphi_t$ is the explicit CP violating angle at tree level that appears due to the existence of the $D-$breaking terms. The acceptable parameters of the model are those ensuring that the above stationary point becomes the absolute minimum at zero temperature.

In the cosmological context it is necessary to use the finite temperature effective potential. It is now known that most naturally the effective potential contains small cubic in temperature contribution, and thus, the phase transition is a second order one. After shifting the scalar fields about their expectation values the asymmetric minimum for the second doublet is

$$\phi_{c,2} = \langle \phi_2(r) \rangle = v f(r) e^{i \varphi_t(T,r)}, \quad (88)$$

where

$$f(r) = \begin{cases} 
0 & (r \leq r_{DW}) \\
\sqrt{1 - \left( \frac{T(c)}{T_W} \right)^2} & (r > r_{DW})
\end{cases}, \quad (89)$$
is a form-function of the wall and has a value from zero to one.

In order to define a width for our domain wall $d_{DW}$ in this configuration of the Higgs vev, we have to define the value of $f(r)$ at the end of the wall. Thus, we will introduce a parameter $\xi$ that relates $d_{DW}$ with the radius of the symmetric region $r_{DW}$. Setting $T(r_{DW}) = T_W$ in Eq. (183), we find

$$d_{DW} = \xi r_{DW} = \xi \frac{9}{256\pi^2} \frac{1}{\beta_{br}} [1 - (T_{br}/T_W)^3]^{-1} \frac{T_{BH}^2}{T_W^2}. \quad (90)$$

We are going to distinguish two cases: $\xi = 1$ which corresponds to an end value $f = 0.6$, and $\xi = 10$ which corresponds to an end value $f = 0.9$.

The structure of the electroweak domain wall is determined only by the thermal structure of the black hole and not by the dynamics of the phase transition as in the ordinary electroweak baryogenesis scenario (the CKN model). In the following subsections we are going to discuss two conditions ensuring that the LTE is valid. The first constraint assumes the size of the domain wall to be greater than the MFP, $1 < d_{DW}/\lambda_s(T_W)$. The second constrains the black-hole lifetime to be large enough to keep the stationary electroweak domain wall, $1 < \tau_{BH}/\tau_{DW}$. Both these constraints refer to the case without accretion. From now on we set $T_{br}$ practically to zero, which is the most logical case.

### 4.3 First Constraint from Thermalization Condition without accretion

The stationary local thermal equilibrium assumption for the scalar wall is valid when the size of the wall is greater than the MFP. Thus, the first constraint is $1 < d_{DW}/\lambda_s(T_W)$, which gives

$$\left(\frac{3}{16\pi}\right)^2 \frac{\xi}{\beta_{s} \gamma} \frac{T_{BH}^2}{T_W^2} > 1, \quad (91)$$

where $\gamma = 1 - (T_{br}/T_W)^3$. In the present study it is more important to find constraints on the black hole mass. Using further the modified formula for the mass of the black hole in five dimensions

$$m_{BH} = \frac{3}{32\pi} \frac{m_5^3}{T_{BH}^2}, \quad (92)$$

the equivalent constraint is

$$m_{BH} < \frac{\xi}{2} \left(\frac{3}{16\pi}\right)^3 m_5^3 T_W^{-2} (\beta_s \beta \gamma)^{-1}. \quad (93)$$

It is now easy to construct Table 1 with the allowed values of black hole masses for various values of the fundamental Planck scale.
Table 1: Summary of the first constraint on the black hole mass for various values of $m_5$ and for the two domain wall thicknesses.

<table>
<thead>
<tr>
<th>$m_5$</th>
<th>$\xi = 1$</th>
<th>$\xi = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 TeV</td>
<td>$m_{BH} &lt; 1.3$ TeV</td>
<td>$m_{BH} &lt; 13$ TeV</td>
</tr>
<tr>
<td>100 TeV</td>
<td>$m_{BH} &lt; 10.6$ TeV</td>
<td>$m_{BH} &lt; 106$ TeV</td>
</tr>
<tr>
<td>10000 TeV</td>
<td>$m_{BH} &lt; 1.06 \times 10^4$ TeV</td>
<td>$m_{BH} &lt; 1.06 \times 10^5$ TeV</td>
</tr>
<tr>
<td>5000 TeV</td>
<td>$m_{BH} &lt; 1.33 \times 10^6$ TeV</td>
<td>$m_{BH} &lt; 1.33 \times 10^7$ TeV</td>
</tr>
<tr>
<td>100000 TeV</td>
<td>$m_{BH} &lt; 1.06 \times 10^8$ TeV</td>
<td>$m_{BH} &lt; 1.06 \times 10^9$ TeV</td>
</tr>
</tbody>
</table>

4.4 Second Constraint from BH Lifetime without accretion

The mean velocity of the outgoing diffusing particles at radius $r_{DW}$ is

$$v_{DW} = \frac{J(r_{DW})}{\rho(r_{DW})} = \left(\frac{32\pi}{9}\right)^2 \beta^2 \left[1 - \left(\frac{T_{br}}{T_W}\right)^3\right]^2 \left(\frac{T_W}{T_{BH}}\right)^2. \quad (94)$$

The characteristic time scale for the construction of the stable electroweak domain wall [44] is

$$\tau_{DW} \simeq \frac{r_{DW}}{v_{DW}} \simeq \frac{729}{262144\pi^4} \frac{1}{\beta^3 T_{BH}^4}. \quad (95)$$

The black hole lifetime needs to be estimated. Assuming that in the black hole dominated universe black holes are spatially separated enough so to neglect accretion among them, we get [49]

$$\frac{dn_{BH}}{dt} \simeq -g_{*SM} \tilde{\sigma}_4 A_{eff,4} T^4 - g_{bulk} \tilde{\sigma}_5 A_{eff,5} T^5, \quad (96)$$

where $\tilde{\sigma}_4$ and $\tilde{\sigma}_5$ are the 4-dim and 5-dim Boltzmann constants per degree of freedom respectively, $A_{eff,4} = 4\pi r_{eff,5}^2$, $A_{eff,5} = 2\pi^2 r_{eff,5}^3$, and $r_{eff,5} = 2r_{BH}$ is the effective black hole radius for black body emission. Neglecting now the evaporation to bulk [62], the black hole lifetime is

$$\tau_{BH} = t_{evap} \simeq \tilde{g}^{-1} \frac{l_4}{l_4} \left(\frac{m_{BH}}{m_4}\right)^2 t_4, \quad (97)$$

where $m_4$ is the 4-dim Planck mass, $l_4$ the 4-dim Planck length, $t_4$ the 4-dim Planck time and

$$\tilde{g} \simeq \frac{1}{160} g_{*SM} + \frac{9}{32\pi^4} g_{bulk}. \quad (98)$$
We have assumed that the degrees of freedom on the thermalised region of the brane are practically the same with the SM $g_*^{SM} = 106.75$ because of the high temperature of the black hole, while $g_{bulk}$ is very small and can be ignored.

For the mechanism to be viable, the black hole lifetime should be larger than the time for the domain wall construction. Thus, the second constraint $1 < \tau_{BH}/\tau_{DW}$ has to be respected, which gives

$$m_{BH} > g_{\ast}^{1/4} \frac{6561^{1/4}}{2^7 \pi^{3/2}} (\beta \gamma)^{-3/4} m_5^{9/4} T_W^{-5/4}.$$  (99)

This black hole mass refers to the initial black hole mass created, while $t_{evap}$ in Eq. (97) is the time for the complete evaporation of this initial black hole mass. The bound of the first constraint also refers to the initial black hole mass. The above constraint is the most strict one. It can be naturally relaxed if the black hole is allowed to accrete plasma from its neighborhood. This will be studied in the next section.

For various values of the 5-dim Planck mass the black hole mass is bounded from below, as shown in Table 2.

<table>
<thead>
<tr>
<th>$m_5$</th>
<th>Black hole mass bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 TeV</td>
<td>$m_{BH} &gt; 42.9$ TeV</td>
</tr>
<tr>
<td>100 TeV</td>
<td>$m_{BH} &gt; 204$ TeV</td>
</tr>
<tr>
<td>1000 TeV</td>
<td>$m_{BH} &gt; 3.6 \times 10^4$ TeV</td>
</tr>
<tr>
<td>5000 TeV</td>
<td>$m_{BH} &gt; 1.35 \times 10^6$ TeV</td>
</tr>
<tr>
<td>10000 TeV</td>
<td>$m_{BH} &gt; 6.45 \times 10^6$ TeV</td>
</tr>
</tbody>
</table>

Table 2: Summary of the second constraint on the black hole mass for various values of $m_5$.

4.5 Efficient Baryogenesis without accretion

Now, we are going to estimate first the baryonic number created by a single black hole, and then the cosmic baryon to entropy ratio $b/s$. The demand $b/s \simeq 10^{-10}$ provides a strict test for all baryogenesis mechanisms.

The sphaleron process works in all the symmetric region and the domain wall. However, the baryon asymmetry production happens on the domain wall where both CP violation and non-equilibrium conditions exist. Furthermore, we want $f(r) = |\langle \phi_2(r) \rangle|/v \leq$
\( \epsilon = 1/100 \) in order that the exponential factor in the sphaleron process be of order one (otherwise the baryon asymmetry would be suppressed). This means that the effective region of baryon generation is the region of the domain wall with small values of the Higgs scalar. The working region that produces baryons is from \( r_{DW} \) till \( r_{DW} + d_{sph} \). Then, \( d_{sph} \) is defined from \( f(r_{DW} + d_{sph}) = \epsilon \). One now can see that \( \int_{r_{DW}}^{r_{DW} + d_{sph}} dr \frac{d}{dr} \varphi(r) = \epsilon \Delta \varphi_{CP} \), where \( \varphi(r, T) = [f(r) - 1] \Delta \varphi_{CP} \) [43]. Thus,

\[
\dot{B} = V \frac{\Gamma_{sph}}{T_W} \mathcal{N} \dot{\varphi} \\
= 4\pi \mathcal{N} \kappa \alpha^5_W T_W^3 v_{DW}^2 \int_{r_{DW}}^{r_{DW} + d_{sph}} dr \frac{d}{dr} \varphi(r) \\
= \frac{1}{16\pi} \mathcal{N} \kappa \alpha^5_W \epsilon \Delta \varphi_{CP} \frac{T_{BH}^2}{T_W} \\
= AT_{BH}^2, \tag{100}
\]

where \( \Gamma_{sph} \) is the sphaleron transition rate, \( \Delta \varphi_{CP} \) the net CP phase, and we have set for convenience \( A = \frac{1}{16\pi} \mathcal{N} \kappa \alpha^5_W \epsilon \Delta \varphi_{CP} \frac{1}{T_W} \). The black hole temperature can be expressed as a function of its lifetime [49]

\[
T_{BH} = \sqrt{\frac{3}{32\pi} \tilde{g}^{-1/4} m_5^{3/4} t_{evap}^{-1/4}}. \tag{101}
\]

If we substitute \( T_{BH} \) in Eq. (202) we get

\[
\dot{B} = A \frac{3}{32\pi} \tilde{g}^{-1/2} m_5^{3/2} t_{BH}^{-1/2} \\
= \tilde{A} (t_{evap,0} - t)^{-1/2}, \tag{102}
\]

where \( \tilde{A} = A \frac{3}{32\pi} \tilde{g}^{-1/2} m_5^{3/2} \) and \( t_{evap,0} \) is the time length for complete evaporation of the initial black hole mass. The time \( t \) runs from 0 (black hole creation) to \( t_{evap,0} \), and the baryon number created by a black hole in its lifetime is

\[
B = \int_0^{t_{evap,0}} \dot{B} dt \\
= 2\tilde{A} t_{evap,0}^{1/2}. \tag{103}
\]

Using Eq. (97) to substitute \( t_{evap,0} \) in terms of the initial black hole mass \( m_{BH} \), and \( l/l_4 = (m_4/m_5)^3 \) we find

\[
B = 2\tilde{\dot{\Lambda}} \tilde{g}^{-1/2} m_{BH} \left( \frac{t_{evap,0}}{l_4} \right)^{1/2} \\
= 2\tilde{\dot{\Lambda}} \tilde{g}^{-1/2} m_5^{-3/2} m_{BH} \\
= \frac{3}{(16\pi)^2} \mathcal{N} \kappa \alpha^5_W \tilde{g}^{-1} T_{BH}^{-1} \epsilon \Delta \varphi_{CP} m_{BH}. \tag{104}
\]
Finally, the total baryon number density created from all black holes is \( b = B n_{BH} \), where 
\( n_{BH} = \frac{\rho_{BH} \, m_{BH}}{m_{BH}} \) is the number density of the black holes assuming a monochromatic spectrum.

In our scenario the universe is black hole dominated. Soon after the black holes complete evaporation, the universe is reheated. Therefore, the cosmic black hole density just before the final stage of very rapid evaporation is almost equal to the cosmic density radiation of the reheated plasma after the end of the evaporation

\[
\rho_{BH}(t_{reh}) \simeq \rho_{rad}(t_{reh}) = \frac{\pi^2}{30} g_{reh} T_{reh}^4. \tag{105}
\]

The entropy density is given by \( s = \frac{2\pi^2}{45} g_{reh} T_{reh}^3 \) [59], where \( g_{reh} \) gives the massless degrees of freedom of the reheated plasma in the asymmetric phase. We choose \( T_{reh} = 95 \) GeV in order to avoid the washing out of the produced baryon asymmetry \( (T_{reh} < T_W \simeq 100 \) GeV). In our study we have the freedom to select the reheating temperature in contrast to [44], where the reheating temperature was fixed using a dubious estimate of the black hole energy density. Thus, the baryon-to-entropy ratio is

\[
\frac{b}{s} = \frac{9}{(32\pi)^2} N_{K_0^5} \bar{y}^{-1} T_{reh} \frac{T_{reh}}{T_W} \Delta \varphi_{CP}. \tag{106}
\]

Notice that the value of the baryon-to-entropy ratio depends neither on \( m_5 \) nor on \( m_{BH} \). For some indicative values of \( \Delta \varphi_{CP} \) the baryon-to-entropy values are

\[
\Delta \varphi_{CP} = \pi \Rightarrow \frac{b}{s} = 1.2 \times 10^{-10}
\]
\[
\Delta \varphi_{CP} = 0.1 \Rightarrow \frac{b}{s} = 4 \times 10^{-12}. \tag{107}
\]

The produced baryon-to-entropy value gets close to the observed \( b/s = 6 \times 10^{-10} \), but only for the maximum and not likely \( \Delta \varphi_{CP} = \pi \).

4.6 Baryogenesis and constraints with accretion

In this section we will investigate the role of accretion first to successful and efficient baryogenesis, and second to the realization of the existence of a black hole dominated era. We assume that black holes, after their formation, not only emit but also absorb radiation from their neighborhood. At the high energy regime of the RS universe, accretion is intense, and so it is expected to result to a period that the whole density of the universe is equal or close to that of the black holes. Later on, during the cosmic evolution, evaporation starts to be more significant and finally the black holes annihilate, reheating the universe.
A phenomenological way to handle accretion is to introduce an effective factor \( f > 1 \) which denotes how much longer the lifetime of the black hole becomes

\[
\tau_{BH} = f \tilde{g}^{-1} m_5^{-3} m_{BH}^2. \tag{108}
\]

Now, the produced baryon number is modified to

\[
B = \frac{3f^{1/2}}{(16\pi)^2} \mathcal{N} \kappa \alpha_w^5 \tilde{g}^{-1} T_w^{-1} \epsilon \Delta\varphi_{CP} m_{BH} \tag{109}
\]

and \( b/s \) of Eq. (106) is multiplied by \( f^{1/2} \). In order to have \( b/s \simeq 6 \times 10^{-10} \), it must be

\[
\Delta\varphi_{CP} = 1 \Rightarrow f \simeq 2 \times 10^2 \\
\Delta\varphi_{CP} = 0.1 \Rightarrow f \simeq 2 \times 10^4 \\
\Delta\varphi_{CP} = 0.01 \Rightarrow f \simeq 2 \times 10^6. \tag{110}
\]

As it will become more clear below, such values of \( f \) can naturally be realized. This is a remarkable result. It is very easy to produce large values of baryon asymmetry and even larger than the required amount, for very small values of \( \Delta\varphi_{CP} \).

Let us discuss at this point the various constraints in the presence of accretion that extends the black hole lifetime. The first constraint Eq. (93) remains intact and refers to the maximum value of black hole mass reached just before evaporation starts to dominate accretion. However, the second bound is modified. It becomes less strict because the black hole lifetime is lengthened. The constrained black hole mass refers to the initial value of the black hole mass

\[
m_{BH,i} > f^{-1/4} \tilde{g}^{1/4} \frac{6561^{1/4}}{2^7\pi^{3/2}} (\beta\gamma)^{-3/4} m_5^{9/4} T_w^{-5/4}. \tag{111}
\]

Table 3 shows some lower black hole mass bounds for some representative combinations of the involved free parameters.

In summary, taking into consideration both constraints and demanding that \( b/s \simeq 6 \times 10^{-10} \), we can find allowed black hole mass ranges for various values of \( m_5 \) and \( \Delta\varphi_{CP} \), i.e., for \( m_5 = 50 \text{ TeV} \) and \( \Delta\varphi_{CP} = 0.01 \) the allowed range is \( 1.1 \text{ TeV} < m_{BH} < 13 \text{ TeV} \), for \( m_5 = 100 \text{ TeV} \) and \( \Delta\varphi_{CP} = 0.01 \) the allowed range is \( 5.3 \text{ TeV} < m_{BH} < 106 \text{ TeV} \), etc. These ranges differ from the previously mentioned case without accretion. There, the estimated ranges show the allowed range of the initial black hole mass. Here, in this section, when accretion is added, the estimated ranges show the allowed wider range of the time dependent black hole mass during the accretion period. Thus, the range \( 5.3 \text{ TeV} < m_{BH} < 106 \text{ TeV} \) means that the initial black hole mass can be as low as \( 5.3 \text{ TeV} \).
and increases during accretion to a value as large as 106 TeV. It is worth mentioning that for smaller values of CP, which is more favorable, the above allowed black hole mass ranges widen!

<table>
<thead>
<tr>
<th>$m_5$ (TeV)</th>
<th>$f$</th>
<th>Initial black hole mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>$2 \times 10^2 (\Delta \varphi_{\text{CP}} = 1)$</td>
<td>$m_{BH,i} &gt; 11 \text{ TeV}$</td>
</tr>
<tr>
<td>50</td>
<td>$2 \times 10^4 (\Delta \varphi_{\text{CP}} = 0.1)$</td>
<td>$m_{BH,i} &gt; 3.5 \text{ TeV}$</td>
</tr>
<tr>
<td>50</td>
<td>$2 \times 10^6 (\Delta \varphi_{\text{CP}} = 0.01)$</td>
<td>$m_{BH,i} &gt; 1.12 \text{ TeV}$</td>
</tr>
<tr>
<td>100</td>
<td>$2 \times 10^4 (\Delta \varphi_{\text{CP}} = 0.1)$</td>
<td>$m_{BH,i} &gt; 17 \text{ TeV}$</td>
</tr>
<tr>
<td>100</td>
<td>$2 \times 10^6 (\Delta \varphi_{\text{CP}} = 0.01)$</td>
<td>$m_{BH,i} &gt; 5.3 \text{ TeV}$</td>
</tr>
</tbody>
</table>

Table 3: Summary of the second constraint on the initial black hole mass for various values of $m_5$ and $f$. The values of $f$, $\Delta \varphi_{\text{CP}}$ are those that give the observed baryon asymmetry ratio.

In the RS model, there is a characteristic transition time $t_c$ that denotes the passage from the high-energy regime with the unconventional Hubble law to the low-energy regime. An interesting and workable case is when accretion is stronger than evaporation and continues till $t_c$, while afterwards evaporation is the dominant term in the differential equations. This case was discussed in [49]. The black hole lifetime is

$$\tau_{BH} = t_c + \frac{1}{\bar{g}} \frac{m_5}{m_{BH,max}} - \frac{1}{\bar{g}} \frac{m_{BH,max}}{m_5} ,$$

(112)

where $m_{BH,max}$ is now the black hole mass at $t_c$. For $m_{BH,max} < m_4$, which is always the case in the present study, it is $\tau_{BH} \simeq t_c$ and so the baryon number produced by a black hole becomes

$$B = 2At_{1/2} = 2A \frac{3}{32\pi} \bar{g}^{-1/2} m_5^{3/2} t_{c}^{1/2} \equiv \frac{3/\sqrt{2}}{(16\pi)^3} \bar{g}^{-1/2} N\kappa\alpha_\epsilon\Delta \varphi_{\text{CP}}^{\text{TW}} \approx m_4 .$$

(113)

Note that the final baryon to entropy ratio does not depend on $m_5$. Some indicative combinations of required CP angles and black holes masses ensuring $b/s \simeq 6 \times 10^{-10}$ are:

$$m_{BH,max} = 10^4 \text{TeV} \Rightarrow \Delta \varphi_{\text{CP}} = 10^{-11}$$
$$m_{BH,max} = 10^7 \text{TeV} \Rightarrow \Delta \varphi_{\text{CP}} = 10^{-14}$$
$$m_{BH,max} = 1 \text{TeV} \Rightarrow \Delta \varphi_{\text{CP}} = 10^{-15} .$$

(114)
The qualitative behaviour of the black hole mass time evolution is very sensitive on the accretion efficiency. If the efficiency is low, the dominant accretion stops inside the high energy regime and the above estimated $m_{BH,max}$ masses decrease.

Here, the first constraint remains the same and refers to the maximum black hole mass. On the other hand, the second constraint is practically always satisfied since now $t_c$ is very large, i.e. for $m_5 = 100$ TeV, $m_{BH} = 10$ TeV, we get $t_c/t_{evap} = 10^{29}$. The second constraint can be estimated from

$$t_{evap} + t_c > \tau_{DW} \Leftrightarrow m_{BH,i}^2 > \frac{729}{262144\pi^4} \frac{2}{\beta^3 \gamma^3} \left( \frac{3}{32\pi} \right)^2 \frac{m_5^9}{m_4^2 T_W^5}.$$ (115)

Since the accretion efficiency, the time of black hole formation and initial black hole mass are unknown quantities, it is not useful to study quantitatively and in full various cases, by solving the differential equation of black hole mass time evolution. However, it becomes apparent that successful baryogenesis can be achieved for very small values of CP angles, which can be also provided from different matter content than the two-Higgs model. Thus, our scenario does not depend on a specific form of the Higgs sector. It only requires a small CP angle on one scalar vev.

Note also that some of the evaporated baryon excess could be eliminated from the same black hole during accretion. This phenomenon is not expected to be significant since the Hawking radiation travels with the escape velocity of the gravitational field. Another possibility is that some of the evaporated baryon asymmetric radiation is eaten by nearby black holes. This complication becomes unimportant by assuming that all black holes are initially widely separated while the expansion further increases the inter black hole distances.

4.7 Black hole domination era due to accretion

Let us discuss now the possibility of a black hole dominant era in the RS setup. This situation can be easily realized due to the strong accretion at the high-energy regime. The differential equation that describes the black hole mass time evolution is

$$\frac{dm_{BH}}{dt} = F \pi r_{eff,5}^2 \rho_{rad} - g_s SM \frac{3\Gamma (4) \zeta (4)}{2^6 \pi^4} \frac{m_5^3}{m_{BH}},$$ (116)

where $F$ is the accretion efficiency and $\rho_{rad}$ is the energy density of the surrounding radiation.
According to [49], if the accretion efficiency factor is $F > 0.78$, the PBH grows (accretion dominates evaporation) until $t_c$ is reached, provided the initial black hole mass is $m_{BH,i} > m_5$. If $F < 0.78$ then the loss due to evaporation is larger than the gain. There is also a case ($m_{BH,i} \gg m_5$ and low efficiency) where we have more accretion than evaporation till “halt” time $t_h$ in the high-energy regime. In radiation dominated high-energy regime it can be proved that $t_h^{1-q} \simeq q \left[1 + (1 - q) \frac{4 F}{\pi} \left(\frac{m_{BH,i}}{m_5}\right)^{3/2}\right] t_i^{1-q}$, where $q = 4F/\pi$ and $\nu$ denotes what fraction of the horizon mass the initial black hole mass comprises.

Demanding a smaller evaporation than accretion, the second term in Eq. (116) becomes suppressed. The differential equation now takes the form

$$\frac{dm_{BH}}{dt} = \frac{2 F}{\pi} m_{BH} t$$

with solution $m_{BH} = m_{BH,i} (t/t_i)^{2F/\pi}$. Let us examine the case where $\rho_{rad}$ initially, at the PBHs’ formation time $t_i$, is much higher than $\rho_{BH}$, i.e. $\rho_{rad,i} = \mu_i n_{BH} m_{BH,i}$ with $\mu_i > 1$. If at the end of accretion period at $t = t_f$ we have a black hole domination, then $\rho_{rad,f} = \mu_f n_{BH} m_{BH,i} (t_f/t_i)^{2F/\pi}$ with $\mu_f < 1$. Energy conservation implies

$$t_f = \left(\frac{1 + \mu_i}{1 + \mu_f}\right)^{\pi/2F} t_i.$$  \hspace{1cm} (118)

This expression implies that it is always possible to start with a radiation dominated era and end in a black hole dominated era within the high-energy regime. This holds, since choosing a small enough value of $t_i$, the time duration $t_f$ can be smaller than $t_h$ or $t_c$, which is the upper bound for the dominant accretion period.

### 4.8 PBHs constraints

The model described in the present work should comply with the constraints coming from observational data. These constraints refer to the fraction of the mass of the universe going into PBHs at formation time, namely they refer to the quantity $\alpha_i = \frac{\rho_{BH,i}}{\rho_{tot}}$. We summarize the possibly relevant observational constraints in relation to the very small PBH masses appearing in our scenario [65], [66], [67], [68], [69], [70]. PBHs with lifetime smaller than $10^{-2}$s are free from BBN constraints because they evaporate well before weak freeze-out and leave no trace. Observation of the extragalactic photon background provides no limit on $\alpha_i$ for very small PBHs as the ones discussed here. For PBHs with masses below $10^4g$, the emitted photons from the evaporation do not violate the observed value of photon-to-baryon ratio. Supersymmetry or supergravity relics provide no limit on $\alpha_i$ for very
small BH masses, so that the observed cold dark matter density is not exceeded. If PBH evaporations leave stable Planck-mass relics, these contribute to the dark matter, and in order not to exceed the critical density there arises an upper bound on $\alpha_i$, but for masses not as small as the ones here. In general, the analysis of all the above constraints has been performed for the standard four-dimensional cosmology, so an appropriate analysis should consider the corresponding corrections due to extra dimensions. To conclude, all the constraints refer to four-dimensional PBHs with masses at least $10^{-5}g$ (created at Planck time $10^{-43}s$). Since our scenario is a higher-dimensional one with a fundamental mass scale of the order of TeV, the allowed PBH masses are of this order, and therefore, it is quite probable that they are too small to be constrained by observational data.

Another general issue regarding accretion of matter into a black hole is the formation or not of shock waves subject to various conditions [71]. A shock is formed when the rotating flow has a high angular velocity that passes the centrifugal barrier. However, even if this velocity is somewhat lower, shocks can also be formed if the pressure of the flow is large. Most literature analyses semi-analytically and numerically this phenomenon in the context of astrophysical black holes. In addition, there are theoretical works assuming newtonian or post-newtonian physics that describe analytically the existence criteria of shock waves. Typically if the angular momentum $l$ is close to the marginally stable value and the initial kinetic energy $e$ for accretion or thermal energy for wind is within a few percent of the rest mass energy, the flow should pass through a shock. One major problem is that for a given set of $e$ and $l$ for every solution that includes a shock there exists another solution which is shock free. Numerical simulations show that if there are significant perturbations in the flow of falling material, more than a certain degree, then there is shock formation. However, these numerical works concern choice of parameters relevant for astrophysical black holes. For primordial black holes generated in a 4-dim FRW universe a crucial criterion is that the perturbation amplitude $\delta$ (defined as the relative mass excess inside the overdense region measured when it had the same scale as the cosmological horizon) is greater than a threshold value $\delta_c$. For perturbation with $\delta$ close to $\delta_c$ numerical calculations reveal that shocks are always formed.

In our context there are primordial black holes embedded in a surrounding cold radiation bath. The radiation temperature during the accretion period needs to be lower than the electroweak scale and it can be very much lower. However, this temperature depends on the specific cosmic scenario that creates the primordial black holes. It is reminded that the universe is reheated after the evaporation of all cosmic primordial black holes. Thus, the falling material needs not to have large kinetic energy. In addition, the
fact that the accretion happens into the RS high energy regime makes the surrounding radiation plasma to be eaten more effectively contrary to the conventional FRW, as the expansion proceeds. The reason is that the slower decrease of the background density during the high-energy regime makes accretion important. Therefore the high energy regime increases the accretion efficiency and not necessarily the speed of the rotating flow of falling material. Nevertheless, a complete study should consider i) the profile type of initial perturbations that created PBHs, ii) the five-dimensional geometry of the black holes, iii) the small black holes masses which make them hot enough to produce significant quantum evaporation, and iv) the complication that the escape of possible shock waves may feed the accretion of nearby black holes, depending on the inter black holes distances and the expansion rate. It worths investigating in a separate work for a certain cosmic scenario of production of brane primordial black holes, the possibility of the formation of shock waves during the accretion period. The study would almost certainly require numerical analysis.

Finally, let us finish mentioning one important point. Although we have performed our analysis of the BH accretion in the high energy regime of a RS cosmology, a similar analysis should also hold for any early cosmology with high energy modifications. Thus, alternative modified gravity models [72] or even braneworld models with high curvature corrections [73] should in principle equally well produce significant baryon asymmetry.

4.9 Black holes mass spectrum

In this section a discussion regarding the effects of a possible initial mass spectrum of primordial black holes is presented. Till now, a monochromatic mass spectrum was assumed. This was necessary in order to be able to find analytical expressions and inequalities and check first if the proposed baryogenesis mechanism works without any conflicts, and second if it is able to generate the required amount of baryon asymmetry.

Let us discuss how the various constraints on the black hole mass are affected from the existence of a black hole mass spectrum. The first constraint that ensures thermalization demands the size of the wall to be larger than the mean free path and this suggests an upper bound on the black hole mass. It is obvious that all the black holes of the spectrum with mass greater than this upper bound are not hot enough to thermalize the surrounding domain wall and thus they do not produce any baryon asymmetry. The exact distribution of the mass spectrum and its upper tail will determine how large or small a correction to the baryon asymmetry will be. It worths as a future work to adapt a specific mechanism
of creation of primordial black holes and analyze numerically the proposed baryogenesis scenario. The second constraint, which comes from demanding the black hole lifetime to be larger than the time scale of stable domain wall construction, generates a lower bound on the black hole mass. Black holes smaller than this limit evaporate too soon. However, as we have explained previously, this constraint in the presence of dominant accretion in the high energy regime becomes extremely weak, since the lifetime of small black holes is considerably extended.

Nevertheless, a not very narrow mass spectrum may modify the calculations of the produced baryon asymmetry. Indeed Eqs. (104), (109), (113) still hold, but now the total baryon number density created from all black holes is given from a more complicated expression

\[ b = \int_0^\infty B N(m, t) \, dm , \]  

(119)

with \( N \) the number density of the mass spectrum of black holes with masses between \( m \) and \( m + dm \). As a general conclusion it suffices to state that the very efficient baryogenesis due to accretion remains unaffected from the presence of mass spectrum. Based on a certain cosmological scenario of creation of PBHs one can estimate the exact baryon asymmetry straightforwardly. More details will follow concerning the relation of the black hole mass spectrum and the time evolution of the scale factor and the cosmic densities.

We are now going to obtain the equations that determine the evolution of the spectrum of primordial black holes, taking full account of either evaporation into radiation or accretion eating radiation, as well as the effect of the black holes on the evolution of the scale factor. It is assumed that the number density of the initial black hole spectrum is described by a power-law form, following [51], [85]. Thus, the initial number density of the primordial black hole spectrum between \( m_0 \) and \( m_0 + dm_0 \) is

\[ N(m_0)dm_0 = A m_0^{-n} \Theta(m_0 - m_c) \, dm_0 \]  

(120)

with \( m_0 = m(t = 0) \) the initial black hole mass. It has been assumed that all the black holes of the mass spectrum form simultaneously at a certain time otherwise analytic results become unnecessarily hard to be obtained. The \( \Theta \) function (with \( \Theta = 1 \) for \( x > 0 \) and \( \Theta = 0 \) for \( x \leq 0 \)) is introduced in order to model the presence of a cut-off mass in the spectrum and protects from the appearance of divergences at low masses limit. The cut-off mass \( m_c \) is natural to be a factor of the fundamental Planck mass, \( m_c = k m_{\text{Pl}} \), with \( k \) an arbitrary dimensionless constant. The power law should be such that the total energy density does not diverge at large masses and this implies \( n > 2 \). However, as Carr notes [51], initial density perturbations in FRW cosmologies that produce primordial black holes
suggest values in the range $2 < n < 3$. In RS cosmology similar ranges for the power law apply [68]. The constant $A$ represents the amplitude of the spectrum and has appropriate units such that $N(m_0) dm_0$ is number density.

Next step is to determine analytically the spectrum $N(m,t) dm$. Both evaporation and accretion modify the value of the cut-off mass (evaporation reduces it). The number density at a given time will be

$$N(t) = \int_0^\infty N(m,t) \, dm , \quad (121)$$

while the energy density is given by

$$\rho_{BH}(t) = \int_0^\infty N(m,t) \, m \, dm . \quad (122)$$

Since the purpose is to evaluate the modifications on the evolution of cosmic densities due to PBHs back-reaction, we are going to distinguish two cases. The first case concerns the description of the cosmic evolution after the evaporation starts to become dominant compared to the accretion. The second case is the description of the cosmic evolution during the era when accretion mainly determines the black hole mass evolution. Any attempt to seek analytical cosmological solutions considering both accretion and evaporation at the same time proved to be non fruitful. However, the most realistic scenario is this that comprises a long dominant accretion time period during the RS high energy regime which ends and is followed by a dominant evaporation era that results to a reheated radiation dominated universe.

4.10 Dominant evaporation era

First we will study the most interesting case when accretion has just stopped to be significant and evaporation dominates the evolution of the black hole mass. The significance of this analysis lies on finding the modifications on the expansion rate that have to be decreasing, allowing the emergence of the conventional radiation expansion law. Accretion has extended the black hole lifetime and thus significant baryogenesis has already been achieved. As soon as evaporation starts to dominate, something that is expected to be certainly true after the high energy regime $t > t_c$, the black hole mass rapidly decreases. The purpose is to estimate deviations on the cosmic densities and scale factor time evolutions.

The black hole mass spectrum has a time evolution first due to the expansion, which will be added later, and second and more physically important due to the evaporation.
Denoting \( m_{BH} \) by \( m \) as above, the rate of loss of a single black hole is given by

\[
\dot{m} = -g_{\text{tot}} \frac{m^3}{m},
\]

(123)

where

\[
g_{\text{tot}} = \frac{1}{2} \left[ \frac{0.0062}{G_{\text{brane}}} g_{SM} + \frac{0.0031}{G_{\text{bulk}}} g_{\text{bulk}} \right] \simeq g_{SM} \frac{3 \Gamma (4) \zeta (4)}{2^6 \pi^4}
\]

(124)

and the second expression disregards the very small bulk contribution in \( g_{\text{tot}} \). The quantities \( G_{\text{brane}}, G_{\text{bulk}} \) represent the grey-body factors for brane and bulk respectively. In the standard cosmology the grey-body factor is equal to 2.6, but precise values are not well known for the braneworld, see discussion in [49]. The effective degrees of freedom \( g_{\text{tot}} \) is a function of temperature. For the time periods referring to the two cases we study in this section, we assume it is a constant. Eq. (123) can now be integrated and gives

\[
m^2 = m_0^2 - 2 g_{\text{tot}} m_5^3 t. \tag{125}
\]

Solving Eq. (125) with respect to \( m_0 \) and differentiating, we are able to find the time evolution of the number density between \( m \) and \( m + dm \) at time \( t \). The time evolved spectrum is

\[
N(m,t)dm = A m^{-n} \left( 1 + \frac{2 g_{\text{tot}} m_5^3 t}{m^2} \right)^{-(n+1)/2} \Theta(m - m_{\text{er}}(t)) \, dm, \tag{126}
\]

where now the cut-off mass has also time evolved and is given by

\[
m_{\text{er}}(t) = k m_5 (1 - 2 g_{\text{tot}} k^{-2} m_5 t)^{1/2}. \tag{127}
\]

It is obvious that after a time \( t_{\text{lim}} = \frac{k^2}{2 g_{\text{tot}} m_5} \) the cut-off mass reaches zero.

The energy per volume that is transferred from the black hole density to the radiation between times \( t \) and \( t + dt \) can be determined from Eq. (122) and is given by

\[
dE = \varrho_{\text{BH}}(t) - \varrho_{\text{BH}}(t + dt) = - \frac{\partial \varrho_{\text{BH}}}{\partial t} \, dt. \tag{128}
\]

The energy density rate can be estimated using the identity

\[
\frac{d}{dx} \int_{g(x)}^{f(x)} h(x,y) \, dy = \int_{g(x)}^{f(x)} \frac{\partial h(x,y)}{\partial x} \, dy + h(x,f(x)) \frac{df(x)}{dx} - h(x,g(x)) \frac{dg(x)}{dx}, \tag{129}
\]

thus we find

\[
\frac{dE}{dt} = - \frac{d}{dt} \int_0^\infty N(m,t) \, m \, dm \tag{130}
\]

\[
= A g_{\text{tot}} (n + 1) m_5^3 \int_{m_{\text{er,max}}}^\infty m^{-n-1} \left( 1 + \frac{2 g_{\text{tot}} m_5^3 t}{m^2} \right)^{-(n+3)/2} \, dm
\]

\[
- A g_{\text{tot}} k^{-n} m_5^{-n+3} (1 - 2 g_{\text{tot}} k^{-2} m_5 t) \, \Theta \left( \frac{k^2}{2 g_{\text{tot}} m_5} - t \right), \tag{131}
\]

45
where
\[ m_{c,\text{max}}(t) = \max[0, m_c(t)]. \] (132)

The first term in Eq. (131) expresses the evolution of the spectrum and is the only non-zero term at late times. The second part of Eq. (131) arises due to the time evolution of the mass cut-off. It is apparent that for times larger than \( t_{\text{lim}} \), the lightest black holes completely evaporate and the \( \Theta \) function causes this term to vanish. Now it is possible to write the full equations of motion describing the expansion. We define for convenience the scale factor at \( t = 0 \) to be one, \( a(t = 0) = 1 \), where \( t = 0 \) corresponds to the time of primordial black holes formation. In all quantities calculated so far the dilution from expansion will have to be added, i.e. the spectral amplitude \( A \) becomes \( A a^{-3} \), the comoving density is \( \rho_{\text{BH}} = \varrho_{\text{BH}} a^{-3} \), and the comoving energy is \( E_{\text{com}} = E a^{-3} \). Since the case under study concerns the dominant evaporation regime which most naturally starts after the high energy regime of the RS cosmology, the set of equations is the following
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3 m_i^4} (\rho_{\text{rad}} + \rho_{\text{BH}}) \] (133)

and
\[
\dot{\rho}_{\text{rad}} = -4 \frac{\dot{a}}{a} \rho_{\text{rad}} + \frac{dE_{\text{com}}}{dt}. \] (134)

Note that we have assumed that black holes exert unimportant kinetic pressure. Furthermore, for late times \( t > t_{\text{lim}} \)
\[
\rho_{\text{BH}} = \frac{1}{a^3} \int_0^\infty N(m, t) m \, dm \\
= \frac{A}{a^3} \int_0^\infty m^{-n+1} \left( 1 + \frac{2 g_{\text{tot}} m_5^3 t}{m^2} \right)^{-\frac{n+1}{2}} \, dm. \] (135)

It is convenient to set \( L = 2 g_{\text{tot}} m_5^3 \) and \( \mu = \frac{m}{\sqrt{L} t} \). Now it is possible to estimate the integral
\[
\rho_{\text{BH}} = \frac{A}{a^3} (L t)^{-\frac{n+2}{2}} \int_0^\infty \mu^{-n+1} \left( 1 + \frac{1}{\mu^2} \right)^{-\frac{(n+1)}{2}} d\mu \\
= \frac{A}{a^3} (2 g_{\text{tot}} m_5^3 t)^{-\frac{n+2}{2}} \frac{\sqrt{\pi}}{4} \frac{\Gamma(-1 + \frac{n}{2})}{\Gamma(\frac{1+n}{2})}, \] (136)

which holds for \( n > 2 \). We observe that it became possible to find the power of the time evolution of the black hole density \( \rho_{\text{BH}} \propto t^{\frac{n+2}{2}} \). It depends on the spectral index \( n \) which most expectedly takes values \( 2 < n < 3 \). The comoving transfer rate per volume \( \frac{dE_{\text{com}}}{dt} \)
for $t > t_{\text{lim}}$ is given by

$$
\frac{dE_{\text{com}}}{dt} = \frac{A}{a^3} (n + 1) g_{\text{tot}} m_5^3 \int_0^\infty m^{-n-1} \left( 1 + \frac{2 g_{\text{tot}} m_5^3 t}{m^2} \right)^{-(n+3)/2} dm
$$

$$= \frac{A}{a^3} (L t)^{-n/2} (n + 1) g_{\text{tot}} m_5^3 \int_0^\infty \mu^{-n-1} \left( 1 + \frac{1}{\mu^2} \right)^{-(n+3)/2} d\mu
$$

$$= \frac{A}{a^3} (n + 1) (2 g_{\text{tot}} m_5^3)^{1-\frac{n}{2}} \sqrt{\pi} \frac{\Gamma\left(\frac{n}{2}\right)}{8 \Gamma\left(\frac{3+n}{2}\right)} t^{-\frac{n}{2}}.
\quad (137)
$$

There are deviations in the time evolution of the radiation density compared to the conventional FRW model. Actually, Eq. (134) using (137) can be integrated to

$$\rho_{\text{rad}} = \frac{c_0}{a^3} + \frac{c_1}{a^4} \int a t^{-n/2} dt
\quad (138)
$$

where $c_0$ is an arbitrary constant and

$$c_1 = A (n + 1) (2 g_{\text{tot}} m_5^3)^{1-\frac{n}{2}} \sqrt{\pi} \frac{\Gamma\left(\frac{n}{2}\right)}{8 \Gamma\left(\frac{3+n}{2}\right)}.
\quad (139)
$$

Then, Eqs. (133), (136) give an integro-differential equation for the scale factor

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{\tilde{c}_0}{a^4} + \frac{\tilde{c}_1}{a^4} \int a t^{-n/2} dt + \frac{\tilde{c}_2}{a^5} t^{2-n/2},
\quad (140)
$$

where $\tilde{c}_0$ is an arbitrary constant and

$$\tilde{c}_1 = \frac{8\pi}{3 m_4^2} c_1,
\quad (141)
$$

$$\tilde{c}_2 = \frac{8\pi}{3 m_4^2} A (2 g_{\text{tot}} m_5^3)^{1-\frac{n}{2}} \sqrt{\pi} \frac{\Gamma\left(-1 + \frac{n}{2}\right)}{4 \Gamma\left(\frac{1+n}{2}\right)}.
\quad (142)
$$

Now, Eq. (140) can be converted, after a differentiation, into a Raychaudhuri equation

$$2 a^3 H (2H^2 + \dot{H}) = t^{-n/2} \left( \dot{\tilde{c}}_1 + \dot{\tilde{c}}_2 \frac{2-n}{2} + \tilde{c}_2 H t \right).
\quad (143)
$$

The derived Raychaudhuri equation cannot be solved analytically but it can be shown that

$$a \propto t^{1/2}
\quad (144)
$$

is a solution of Eq. (143) neglecting terms of order $t^{-n/2}$. Thus, for times much after the end of evaporation the usual expansion is recovered. More definite results can be only extracted from numerical calculations and simulations covering various ranges of the involved free parameters.
4.11 Dominant accretion era

Here we will analyse another interesting case. It refers to the time period after primordial black hole creation. Since the creation happens in the high energy regime of RS cosmology it is expected accretion to be much more significant than evaporation. The purpose is to estimate the time evolutions of the cosmic densities and the scale factor.

The black hole mass spectrum has now a time evolution due to the accretion, apart from the expansion which will be added later. The rate of loss of a single black hole is given by

$$\dot{m} = F \pi r_{\text{eff},5}^2 \rho_{\text{rad}} = \frac{32}{3} \frac{m}{m_5^3} \rho_{\text{rad}},$$

(145)

where the $\rho_{\text{rad}}$ represents the surrounding to the black holes radiation density. The time duration of this case where accretion is dominant is much longer than the regime of dominant evaporation. Since most baryon asymmetry is produced during this accretion period it worths describing the complicated equations of motion. Eq. (145) can be solved and gives

$$m = m_0 \zeta \exp \left( \int_0^t \rho_{\text{rad}} dt \right),$$

(146)

where $\zeta = \exp(\frac{32F}{3m_5^3})$.

Solving Eq. (146) with respect to $m_0$ and differentiating, we are able to find the time evolution of the number density between $m$ and $dm$ at time $t$, with the help of Eq. (120). The time evolved spectrum now is

$$N(m,t)dm = A \zeta^{n-1} \exp \left[ (n-1) \int_0^t \rho_{\text{rad}} dt \right] m^{-n} \Theta(m - m_{ca}(t)) dm,$$

(147)

where the cut off mass has been time evolved from $m_c$ to $m_{ca}$ given by

$$m_{ca}(t) = k m_5^5 \zeta \exp \left( \int_0^t \rho_{\text{rad}} dt \right).$$

(148)

It is obvious that contrary to the previous case the cut off mass does not equal zero at any time.

The energy per volume that is transferred from the eaten radiation to the black hole density between times $t$ and $t + dt$ can be determined from $dE = \varrho_{BH}(t) - \varrho_{BH}(t + dt) = \frac{\partial \varrho_{BH}}{\partial t} dt$ and the energy density rate can be estimated using in addition Eq. (129). Thus

$$\frac{dE}{dt} = A \frac{n+1}{n-2} \zeta k^{-n+2} m_5^{-n+2} \exp \left( \int_0^t \rho_{\text{rad}} dt \right) \rho_{\text{rad}}$$

(149)

$$- A \zeta k^{-n+2} m_5^{-n+2} \exp \left( \int_0^t \rho_{\text{rad}} dt \right) \rho_{\text{rad}} \Theta(m - m_{ca}).$$

(150)
The first term in Eq. (149) expresses the evolution of the spectrum, while the second part arises due to the time evolution of the mass cut-off. In this second case this term does not vanish as long as evaporation is less significant than accretion. Now it is possible to write the full equations of motion describing the expansion. All densities should become comoving multiplying them with $a^{-3}$. In this case of dominant accretion regime we are clearly in the high energy regime of the RS cosmology. Therefore the set of equations is the following

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_4^2} \left(\rho_{\text{rad}} + \rho_{\text{BH}} + \frac{1}{2\lambda}(\rho_{\text{rad}} + \rho_{\text{BH}})^2\right)$$

with $\lambda = \frac{3m_6^2}{4\pi m_4^2}$ and

$$\dot{\rho}_{\text{rad}} = -4\frac{\dot{a}}{a}\rho_{\text{rad}} + \frac{dE_{\text{cm}}}{dt}.$$  

For simplicity the same assumption as before has to be made, i.e. black holes exert unimportant kinetic pressure. Thus, we get

$$\rho_{\text{BH}} = \frac{1}{a^3} \int_{m_5}^{\infty} A \zeta^{n-1} \exp\left((n-1) \int_0^t \rho_{\text{rad}} \, dt\right) m^{-n+1} \, dm$$

$$= \frac{A}{a^3} \frac{1}{n-2} \zeta (k m_5)^{-n+2} \exp\left(\int_0^t \rho_{\text{rad}} \, dt\right).$$

We observe that in order to proceed further and be able to find the power of the time evolution of the black hole density we have to know the integral $\exp(\int_0^t \rho_{\text{rad}} \, dt)$ since $\rho_{\text{BH}}$ is proportional to it. The comoving transfer rate per volume $\frac{dE_{\text{cm}}}{dt}$ is given by

$$\frac{dE_{\text{cm}}}{dt} = \frac{A}{a^3} \frac{n-1}{n-2} \zeta k^{-n+2} m_5^{-n+2} \rho_{\text{rad}} \exp\left(\int_0^t \rho_{\text{rad}} \, dt\right)$$

$$- \frac{A}{a^3} \zeta k^{-n+2} m_5^{-n+2} \rho_{\text{rad}} \exp\left(\int_0^t \rho_{\text{rad}} \, dt\right) \Theta(m - m_5).$$

The complete set of equations Eqs. (151), (152), (153) and (154) form an integro-differential system and can be solved only numerically for various ranges of the parameters.

### 4.12 Conclusions

The present study shows that the proposed baryogenesis scenario of accreting primordial black holes in a RS braneworld is capable to generate efficient baryogenesis even for very small CP violating angles. In summary the key points are
• The allowed by the mechanism BH mass range includes a mass spectrum around the higher dimensional Planck mass. The latter is important since this mass spectrum is energetically favorable to be generated from high energy interactions in the very early braneworld cosmic history.

• The baryogenesis process in a 5-dim RS cosmology becomes easier than in the standard 4-dim universe because of the accretion in the high energy regime.

• The Higgs sector has not to be necessarily that of the two-Higgs model. It just requires a Higgs sector with very low CP asymmetry.

• It is not necessary the universe to be BH dominated at the time of the BHs creation since it is possible to turn into BH domination due to the accretion. However, since the proposed mechanism is able to generate very large baryon asymmetry, the black hole domination requirement is not crucial.

• The key point of producing large baryon asymmetry is the existence of an early high energy regime with an unconventional expansion rate that favors accretion. Thus, any alternative cosmological model bearing this feature can also give efficient baryogenesis.
5 Brans-Dicke universe

Brans-Dicke theory \cite{75} is another modified gravity model that exhibits a very early high energy era. Primordial black holes that live in this era are expected to accrete intensely radiation from their neighborhood. This way their lifetime is considerably elongated and also it is possible for them to dominate the universe. Both contribute to increased baryon number-creation by black holes emission.

5.1 Brans-Dicke gravity

The gravitational field of General Relativity is a tensor of the second rank. Additional gravitational fields, though, could exist in the form of scalars, vectors, tensors or even higher rank fields. Of course, these should not contradict observational and experimental data. See \cite{76} for a review of gravities with extra fields (and modified gravities in general). The simplest extra field is a scalar field and the simplest case is Brans-Dicke gravity. One motivation for studying and using scalar-tensor gravities during the last two or three decades is that they appear as the 4-dimensional effective theories of gravitational models with extra dimensions.

The Lagrangian of a scalar-tensor gravity theory can be written as \cite{77}

\[ L = -\phi R + \phi^{-1}\omega(\phi)\partial_\mu\phi\partial^\mu\phi + 16\pi L_m, \]  

(155)

where \( \phi \) is the scalar field, \( \omega(\phi) \) is the coupling function and \( L_m \) is the Lagrangian of the matter fields. For \( \omega = \text{constant} \) we have the Brans-Dicke theory.

From the Lagrangian above and the associate action we can have the generalized Einstein equations and \( \phi \) wave equation: the first by varying action with respect to the metric and the second with respect to \( \phi \). So, the generalized Einstein equation is

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi\phi^{-1}T_{\mu\nu} - \omega(\phi)\phi^{-2}(\phi_\mu\phi^\mu - \frac{1}{2}g_{\mu\nu}\phi^i\phi^i) - \phi^{-1}(\phi_{\mu\nu} - g_{\mu\nu}\Box\phi), \]

(156)

where \( \phi_\mu = \partial_\mu \phi \), \( \Box = g^{\mu\nu}\partial_\mu \partial_\nu \) and \( T_{\mu\nu} \) is the energy-momentum tensor of the matter. One can see that for \( \phi = \text{constant} \) all of equation’s second part vanish, except the first one. Thus, General Relativity Einstein equation is retrieved, with \( G = \phi^{-1} \).

\( \phi \) wave equation is

\[ (3 + 2\omega(\phi))\Box\phi = 8\pi T - \frac{d\omega}{d\phi}\phi^i\phi_i. \]

(157)

For \( \phi = \text{constant} \) trace \( T \) has to be zero and this is so for radiation fluid. Hence, solutions of General Relativity for radiation dominated universe \((T = 0)\) are also solutions of the
5.2 Brans-Dicke Cosmology

We suppose that universe is homogeneous and isotropic and also that contains a perfect fluid with equation of state \( p = \gamma \rho \). Then, the modified Einstein equations give modified Friedman equations [78]:

\[
H^2 + \frac{\dot{\phi}}{\phi} - \frac{\omega \dot{\phi}^2}{6\phi^2} + \frac{k}{a^2} = \frac{8\pi \rho}{3\phi},
\]

\[
\ddot{\phi} + \left( \frac{3H + \frac{\omega}{2\omega + 3}}{2\omega + 3} \right) \dot{\phi} = \frac{8\pi \rho}{2\omega + 3},
\]

\[
\dot{H} + H^2 + \frac{\omega \dot{\phi}^2}{3\phi^2} - \frac{H^2}{\phi} = -8\pi \rho \frac{(3\gamma + 1)\omega + 3}{3\phi(2\omega + 3)} + \frac{\dot{\omega} \phi}{2\phi(2\omega + 3)},
\]

\[
\dot{\rho} + 3(\gamma + 1)H\rho = 0.
\]

We remind that \( a \) is the scale factor, \( H = \dot{a}/a \) is the Hubble constant and \( k \) is the curvature constant. Generally, we assume \( k = 0 \), since this seems to approximately agree with observation. Setting \( \omega = constant \) for Brans-Dicke model the above equations become:

\[
\frac{\ddot{a}}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} - \frac{\omega \dot{\phi}^2}{6\phi^2} = \frac{8\pi \rho}{3\phi},
\]

\[
2\ddot{a} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} + \frac{\omega \dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi \rho}{\phi},
\]

\[
\frac{\ddot{\phi}}{8\pi} + 3\frac{\dot{a}}{a} \frac{\dot{\phi}}{8\pi} = \frac{\rho - 3p}{2\omega + 3}.
\]

These equations have been solved for various cases and phases of the universe. What interests us can be found in [83], [84], [85]. In Brans-Dicke cosmology, in contrast to General Relativity, there is a very early vacuum domination era. The exact vacuum solution for Brans-Dicke, that is \( \omega = constant \) is:

\[
G = \phi^{-1} \propto t^{-\frac{a}{1+d}},
\]

\[
a \propto t^{-3(1+d)}
\]

where

\[
d = \omega^{-1} \left(1 + \sqrt{1 + \frac{2\omega}{3}} \right).
\]
Vacuum domination lasts until some time $t_1$. For $t > t_1$ it is radiation dominated, as in General Relativity, until $t_{eq} \sim 10^{11}$ s. $t_{eq}$ is the time that matter density equals radiation density. We remind now (Eq. (156)) that the General Relativity solutions for the radiation dominated era are also Brans-Dicke solutions. So

$$G = \phi^{-1} = \text{constant}, \quad (165)$$

$$a \propto t^{1/2}.$$ 

After $t_{eq}$, matter is the dominant ingredient of the universe and $G$ varies again:

$$G = \phi^{-1} \propto t^{-n}, \quad (166)$$

$$a \propto t^{2-n},$$

where

$$n = \frac{2}{4 + 3\omega}. \quad (167)$$

Solar system observations indicate a very large $\omega \gtrsim 10000$ and equivalently a very small $n$ [86]. The expressions then of $G$ and $a$, using also $n$ instead of $d$, become:

$$G(t) \simeq G_0(t_0/t)^{\sqrt{n}(t_0/t_{eq})^n}, \text{ if } t < t_1:\ BD - \text{field dominated}$$

$$G_0(t_0/t_{eq})^n, \text{ if } t_1 < t < t_{eq}: \text{radiation dominated}$$

$$G_0(t_0/t)^n, \text{ if } t_{eq} < t: \text{dust dominated} \quad (168)$$

and

$$\alpha(t) \propto t^{(1-\sqrt{n})/3}, \ BD - \text{field dominated}$$

$$t^{1/2}, \text{ radiation dominated}$$

$$t^{(2-n)/3}, \text{ dust dominated} \quad (169)$$

where $t_0$ is the present time and $G_0$ is the present value of $G$. We note here that, up till now, we have not considered a black hole domination era, which is very significant for our model. We will do so at the presentation of our model.

### 5.3 Black holes in Brans-Dicke universe

Primordial black holes may form at a very early time from inhomogeneities. Nevertheless, our model of baryogenesis by primordial black holes in a universe with Brans-Dicke gravity presented in the next section, is independent of the black holes creation mechanism.
The mass evolution of a black hole is determined by accretion and evaporation. A black hole emits Hawking radiation and thus mass. The mass loss rate is proportional to the fourth power of black holes temperature, so, while it may be extremely low for astrophysical black holes, it is very intense for the tiny primordial black holes that are suitable for our baryogenesis model.

\[
\dot{M}_{ev} = -4\pi R_{BH}^2 a_H T_{BH}^4 = -\frac{a_H}{256\pi^3 G^2 M_{BH}^2}, \tag{170}
\]

where \( a_H \) is the Stefan-Boltzmann constant. Also we have used the black hole radius

\[
R_{BH} = 2GM_{BH}, \tag{171}
\]

and the relation between black hole mass and temperature

\[
T_{BH} = \frac{1}{8\pi G M}. \tag{172}
\]

A black hole is expected to accrete whatever matter falls into it and this way there is an increase of its mass. Black hole mass gain rate due to accretion is proportional to the black hole surface and also to the surrounding matter density \( \rho \):

\[
\dot{M}_{acc} = 4\pi f R_{BH}^2 \rho, \tag{173}
\]

where \( f \) is the accretion efficiency. It is \( f \sim O(1) \) and is rather less than 1. Combining both accretion and evaporation, black hole mass change rate becomes

\[
\dot{M}_{BH} = 4\pi f R_{BH}^2 \rho - \frac{a_H}{256\pi^3 G^2 M_{BH}^2}. \tag{174}
\]

We are going to use this equation in our model of baryogenesis, but not the solutions derived by other researchers, since our assumptions on universe evolution are different. Instead, we solve it in the frame of our scenario (see next chapter).

**Black hole mass evolution during BD-field domination**

Mass evolution of primordial black holes in Brans-Dicke universe considering both evaporation and accretion is studied in [80]. Accretion is not effective during the \( \phi \)-field domination epoch \( (t < t_1) \) and so we can only consider evaporation. Equation (200) becomes

\[
\dot{M}_{ev} = -\frac{a_H}{256\pi^3 G_0^2} \left( \frac{t_{eq}}{t_0} \right)^2 \left( \frac{1}{t_1} \right)^{2\sqrt{\pi}} \left( \frac{t_{eq}}{M_{BH}^2} \right). \tag{175}
\]
We integrate and the solution found is

\[ M_{3BH}^3 = M_i^3 + \frac{3a_H}{256\pi^4G_0^2} \left( 1 + 2\sqrt{n} \right)^{n-1} \left( \frac{t_{eq}}{t_0} \right)^{1/n} \left( t_1 \right)^{-2\sqrt{n}} \left( t_{1}^{2\sqrt{n}+1} - t_{1}^{2\sqrt{n}+1} \right) . \]  \hspace{1cm} (176)

The black hole lifetime, if it is small enough to evaporate before \(\phi\)-domination ends, is [78]

\[ \tau = \left( \frac{256\pi^3G_0^2}{3a_H} \right) \left( 1 + 2\sqrt{n} \right)^{n} \left( \frac{t_0}{t_{eq}} \right)^{1/n} M_i^{3\sqrt{n} \left( t_{eq}/t_0 \right)} + t_{i}^{1+2\sqrt{n}} \right] \frac{1}{1+2\sqrt{n}} . \]  \hspace{1cm} (177)

**Black hole mass evolution during radiation domination**

Black holes created during the \(\phi\)-domination period may survive after it. In this case, the mass of the black hole at the transition time \(t_1\) from \(\phi\) domination to radiation domination, is calculated by equation (176)

\[ M_{BH}^3(t_1) = M_i^3 + \frac{3a_H}{256\pi^4G_0^2} \left( 1 + 2\sqrt{n} \right)^{n} \left( \frac{t_{eq}}{t_0} \right)^{1/n} \left( t_1 \right)^{-2\sqrt{n}} \left( t_{1}^{2\sqrt{n}+1} - t_{1}^{2\sqrt{n}+1} \right) . \]  \hspace{1cm} (178)

During radiation-dominated era now \(t_1 < t < t_{eq}\), it is

\[ \dot{M}_{BH} = \frac{3}{2} fG_0 \left( \frac{t_0}{t_{eq}} \right)^{n} \left( \frac{M_{BH}^2}{t^2} \right) - \frac{3a_H}{256\pi^4G_0^2} \left( \frac{t_{eq}}{t_0} \right)^{1/n} \left( \frac{t_{eq}}{t_0} \right)^{2n} \frac{1}{M_{BH}^2} . \]  \hspace{1cm} (179)

This equation can be solved numerically. In the case when the primordial black hole was born at a time \(t_i > t_1\) with a mass \(M_i\), then these are the initial conditions for the solution. If the black hole was born before \(t_1\), then \(t_1\) and \(M_{BH}(t_1)\) will be used as initial conditions for the solution during the radiation dominated epoch.
6 Baryogenesis by primordial black holes in Brans-Dicke cosmology

6.1 Introduction

The present chapter is based on [2]. Here we propose a novel model of electroweak baryogenesis by PBHs in Brans-Dicke cosmology. We assume that the early Universe starts from a small enough primordial black hole dominated era or a mixture of radiation and small enough primordial black holes. Brans-Dicke theories can realize such a scenario. While universe temperature has been lowered below electroweak symmetry breaking point ($\sim 100\text{GeV}$), a region around each PBH is reheated by Hawking radiation to $T > 100\text{GeV}$. A domain wall is formed between the symmetric and asymmetric regions and this is where baryogenesis takes place, by sphaleron processes. Its key characteristics are:

1. The phase transition at the domain wall can be of second order. The baryon over antibaryon excess is created by sphalerons, not destroyed.
2. In order to produce the observed baryon number ($b/s \simeq 6 \times 10^{-10}$), the universe needs to become PBH dominated. In BD - cosmology this may happen naturally, because of accretion by the PBHs. In standard cosmology, on the contrary, it is accepted that accretion cannot be significant [81].
3. The CP-violating angle must be quite large for adequate baryogenesis. This can be satisfied incorporating a realistic two Higgs doublet, instead of a single Higgs, in our model [82].

We assume that PBHs are created at the end of the BD - field ($\phi$) domination era, although the model is not dependent on how they were created. Accretion can lead to BHs mass increase only when there is enough radiation for BHs to accrete. This may happen during radiation domination or even BH - domination, if there is enough radiation density, as we ’ll show. We examine two cases: the first is that the universe becomes BH dominated immediately after PBHs creation, with BH density $\rho_{BH} = 0.7\rho$ and radiation density $\rho_{rad} = 0.3\rho$. The second is the case that PBHs are initially, immediately after their formation, only a small part of the universe but then, because of intense accretion, become dominant. It will be shown that for both cases there is a range of initial PBHs masses for which accretion leads the universe to become completely BH dominated ($\rho_{BH} \simeq 100\%$).

The advantage of the proposed scenario is that Brans-Dicke gravity, due to enhanced accretion, can naturally provide black holes domination in the early Universe and at the same time, as we are going to show, efficient baryogenesis for smaller CP-violating angles.
compared to the case of the same scenario but with the gravity of General Relativity.

In the following section, the baryon asymmetry mechanism is described. In section 3 we analyse the first of the two cases of the proposed scenario, a black hole dominated Universe, while in section 4 we study a Universe that initially is radiation dominated but then becomes black hole dominated. Next a section with various bounds is given and finally the last section provides a conclusive summary.

6.2 Baryon number created by a single primordial black hole

The PBHs of our proposed mechanism are surrounded by radiation colder than the electroweak breaking point \((T_W \sim 100\text{GeV})\). They are very small and thus Hawking temperature \(T_{BH}\) is much greater than this temperature. Then all kinds of Standard Model (SM) particles are emitted and they are in symmetric phase. So, the Hawking emission causes the thermalization of the black hole surrounding region. A local temperature \(T(r)\) can be defined for a region with size greater than the mean free path (MFP) of the emitted particles. The MFP of a particle \(f\) is \(\lambda_f(T) = \frac{\beta_f}{T}\), where \(\beta_f\) is a constant that depends on the particle species. Quarks and gluons have a strong interaction and they have the shortest MFP with \(\beta_s \simeq 10\). Because of the high, \(> T_{EW}\), reheating temperature, all SM particles contribute to the massless degrees of freedom \(g_{*SM} \equiv \sum_f g_{*f} = 106.75\). So, the radiation density is \(\rho = \frac{\pi^2}{30} g_{*SM} T^4(r)\).

Yet the area closest to the PBH horizon externally, with depth the quarks and gluons MFP, is not thermalized. For this reason, the emitted particles move freely there and most of them don’t drop back to the black hole. Thus, the black hole radiation obeys the law of Stefan-Boltzmann with no corrections. Now let \(r_o\) be the minimum thermalized radius and \(T_o\) the local temperature there: \(T_o = \frac{\beta_s}{r_o}\). We consider then the transfer equation of the energy in the thermalized region to determine the temperature distribution \(T(r)\) [44].

We assume diffusion approximation of photon transfer at the deep light-depth region is valid [61]. The diffusion current of energy in Local Temperature Equilibrium (LTE) is \(J_\mu = -\frac{\beta}{3T(\sigma)} \partial_\mu \rho \). \(\beta/T\) is the effective MFP of all particles by all interactions with \(\beta \simeq 100\). The transfer equation is \(\frac{\partial}{\partial t} \rho = -\nabla_\mu J^\mu\). A stationary spherical-symmetric solution [61] is

\[
T(r)^3 = T_{bg}^3 + \frac{r_o}{r} (T_o^3 - T_{bg}^3).
\]

where \(T_{bg}\) is the background temperature. It can be as high as somewhat lower than \(T_{EW}\), where sphaleron rate is suppressed.
The quantities \( r_o \) and \( T_o \) can be written as functions of black holes temperature \( T_{BH} \) by equalizing the outgoing diffusion flux \( 4 \pi r_o^2 J(r) \simeq \frac{8 \pi^3}{15 \sqrt{\beta}} \beta \beta g_{SM} \left[ 1 - (T_{bg}/T_o)^3 \right] T_o^2 \) with the Hawking radiation flux \( 4 \pi r_{BH}^2 \times \frac{\pi^2}{120} g_{SM} T_{BH}^4 \):

\[
\begin{align*}
    r_o &= \frac{16 \pi}{3} \frac{1}{T_{BH}} \sqrt{\beta^3 \beta \left[ 1 - (T_{bg}/T_o)^3 \right]} \\
    \text{and} \\
    T_o &= \frac{3}{16 \pi \sqrt{\beta}} \frac{T_{BH}}{\sqrt{1 - (T_{bg}/T_o)^3}}.
\end{align*}
\]

\( T_{bg} \ll T_o \) and so the spherical thermal distribution surrounding the black hole is

\[
T(r)^3 = T_{bg} + \frac{9}{256 \pi^2 \beta} \frac{T_{BH}^2}{r}
\]

for \( r > r_o \).

As mentioned before, the region around PBHs is reheated to temperatures higher than the electroweak breaking point and so symmetry is restored there. The background temperature, at the same time, remains below the electroweak breaking point and the symmetry broken. That means that an electroweak domain wall forms around the black hole and it starts at \( r_{DW} \). The phase transition at the domain wall does not have to be of first order. It can be a second order transition. This enlarges the parameter space of the validity of our proposed scenario.

Instead of a single Higgs SU(2) doublet, we incorporate a two Higgs doublet model (2HDM) in our model, since it can accommodate both a large CP - violation angle and CP-violation in the Higgs sector. In our study in order to be able to validate our scenario and provide bounds, we use the 2HDM of [82] which is not only a serious model but also a typical example of the models that could comply with our idea. Our scenario needs a non-negligible CP violation in the Higgs finite temperature corrected potential. The tree-level, CP-breaking scalar potential in [82] is

\[
V_{tree} = m^2_{11} \Phi^\dagger_1 \Phi_1 + m^2_{22} \Phi^\dagger_2 \Phi_2 - \left[ m^2_{12} \Phi^\dagger_1 \Phi_2 + h.c. \right] + \frac{1}{2} \lambda_1 ( \Phi^\dagger_1 \Phi_1 )^2 + \frac{1}{2} \lambda_2 ( \Phi^\dagger_2 \Phi_2 )^2 + \lambda_3 ( \Phi^\dagger_1 \Phi_1 ) ( \Phi^\dagger_2 \Phi_2 ) + \lambda_4 ( \Phi^\dagger_1 \Phi_2 ) ( \Phi^\dagger_2 \Phi_1 ) + \left[ \frac{1}{2} \lambda_5 ( \Phi^\dagger_1 \Phi_2 )^2 + h.c. \right],
\]

where

\[
\Phi_1 = \begin{pmatrix} \phi^+_1 \\ \phi^0_1 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi^+_2 \\ \phi^0_2 \end{pmatrix}
\]
are the two $SU(2)_L$ scalar field doublets. One can see that a $Z_2$ discrete symmetry holds, under which $\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$. Because of this symmetry there are no flavour changing neutral currents. The symmetry is softly broken only by $m_{12}$. The parameters of the potential are real, because of its hermiticity, except from the mass parameter $m_{12}$ and the quartic coupling $\lambda_5$. With this scalar potential it is possible the doublets VEVs to be complex and this CP-violation cannot be gauged away due to the complex values of $m_{12}$ and $\lambda_5$. Note that in our previous work [?] a different two Higgs model had been adopted.

We can simplify the form of the doublets with an $SU(2)$ rotation. $\partial V/\partial \phi_i = 0$ solutions give stationary points, including the asymmetric minimum that respects the $U(1)$ of electromagnetism: $\Phi_1 = \frac{1}{\sqrt{2}}(0, u)^T$, $\Phi_2 = \frac{1}{\sqrt{2}}(0, ve^{i\varphi})^T$. where $u, v, \varphi$ are real and $\varphi$ is the CP-violating angle. This tree-level CP-violating phase depends on $m_{12}$ and $\lambda_5$ and cannot be shifted by an $SU(2)$ rotation. However, in this case, we need this CP angle to be very small due to Electron Dipole Moment constraints (EDM). To achieve strong CP-violations one can hope the loop finite temperature corrected potential to result to big CP-violating cases. In this case, the constraints from EDM do not apply if at zero temperature the CP angle goes to very small values.

Regarding the cosmological consequences, anyway, the finite temperature effective potential is this that should be used. The temperature loop corrections incorporate for the larger range of the parameters space only small cubic resulting to a second order phase transition (in [82] the case of first order transition is also studied, something that is not needed in our scenario). We shift the scalar fields about their expectation values and the second doublet asymmetric minimum becomes

$$\Phi_1 = \frac{1}{\sqrt{2}}(0, u(T))^T, \quad \Phi_2 = \frac{1}{\sqrt{2}}(0, v(T)e^{i\varphi(T)})^T,$$

with

$$v(T) = v f(r),$$

where $f(r)$ is a form-function of the wall and has a value from zero to one; $f(r) = 0$ for $r \leq r_{DW}$ and $f(r) = \sqrt{1 - \left(\frac{T(r)}{T_W}\right)^2}$ for $r > r_{DW}$.

At the limit between the thermalized sphere and the domain wall, the temperature is $T(r_{DW}) = T_W$. Setting this in Eq. (183), we find the radius of the thermalized region $r_{DW}$. The width of the domain wall $d_{DW}$ is about of the order of $r_{DW}$.

$$d_{DW} \simeq r_{DW} = \frac{9}{256\pi^2} \frac{1}{\beta_{br} \left(1 - \left(\frac{T_{bg}}{T_W}\right)^3\right)} \frac{T_{BH}^2}{T_W^3}.$$

59
The structure of the electroweak domain wall is determined only by the thermal structure of the black hole and not by the dynamics of the phase transition as in the ordinary electroweak baryogenesis scenario (the CKN model).

### 6.3 Primordial black holes domination from their creation

In our model, the universe at the beginning of its life is dominated by the BD-field. We assume that the PBHs creation happens at about the end of this period. Then the universe becomes a mixture of radiation and black holes. A first scenario we examine is that universe is BH dominated immediately after $t_1$, that is $\rho_{BH} > \rho_{rad}$. It becomes completely BH dominated because of accretion, if their initial masses are above the limit that accretion exceeds evaporation and the radiation is dense enough, as it will be shown. What follows is that having no more radiation to accrete, they only evaporate. $t_{ev}$ is the time of complete evaporation. The universe then turns radiation dominated, with the observed baryon number already produced. Later the universe turns from radiation to dust dominated at $t_{eq}$. It remains dust dominated until now ($t_0$).

Barrow and Carr at [78] have obtained solutions for $G$ for the three different eras of a model where the universe is initially dominated by the BD-field, then it turns radiation dominated and finally dust dominated:

$$G(t) \simeq G_0\left(\frac{t_1}{t}\right)^n\left(\frac{t_0}{t_{eq}}\right)^n, \text{ if } t < t_1 : \text{BD - field dominated}
$$

$$G_0\left(\frac{t_0}{t_{eq}}\right)^n, \text{ if } t_1 < t < t_{eq} : \text{radiation dominated}
$$

$$G_0\left(\frac{t_0}{t}\right)^n, \text{ if } t_{eq} < t : \text{dust dominated}
$$

where $t_1$ is the time of transition from BD-field dominated to radiation and $n = \frac{2}{4+3\omega}$. To avoid confusion it is worth mentioning that there is no PBHs-domination era at the work of Barrow and Carr.

The modified solutions for our model are:
\[ G(t) \simeq G_0 \left( \frac{t}{t_i} \right)^{\sqrt{n} \left( \frac{t_0}{t_{eq}} \right)^n}, \text{ if } t < t_i : \text{BD - field dominated} \]
\[ G_0 \left( \frac{t_0}{t_{eq}} \right)^n, \text{ if } t_i < t < t_{ev} : \text{PBHs dominated} \]
\[ G_0 \left( \frac{t}{t_{eq}} \right)^n, \text{ if } t_{ev} < t < t_{eq} : \text{radiation dominated} \]
\[ G_0 \left( \frac{t}{t_i} \right)^n, \text{ if } t_{eq} < t : \text{dust dominated} \] (190)

Then we need to write formulas for universe density due to PBHs \( \rho_{BH} \) and scale factor \( \alpha \). The number density of PBHs at the time of their creation is:

\[ n_{BH}(t_i) = \frac{\rho_{BH}(t_i)}{m_{BH}(t_i)} \] (191)

BHs can be treated as dust, regarding the universe ’s density due to them. Because of the fact that their mass changes due to accretion and evaporation, it is their number density \( n_{BH}(t) \), not density, that is inversely proportional to scale factor 3rd power, and so:

\[ \rho_{BH}(t) = n_{BH}(t)m_{BH}(t) = n_{BH}(t_i)\frac{\alpha^3(t_i)}{\alpha^3(t)}m_{BH}(t) = \rho_{BH}(t_i)\frac{m_{BH}(t)}{m_{BH}(t_i)}\frac{\alpha^3(t_i)}{\alpha^3(t)} \] (192)

We assume that the number of black holes after their primordial creation and till their evaporation remains the same. Thus, we assume that these PBHs do not ”eat” each other in a considerable rate during the accretion. Accretion concerns the surrounding radiation mainly.

In [78] can be found the scale factor’s time evolution:

\[ \alpha(t) \propto t^{(1-\sqrt{n})/3} \], BD – field dominated
\[ t^{1/2} \], radiation dominated
\[ t^{(2-n)/3} \], dust dominated \] (193)

and so

\[ \rho_{BH}(t) = \rho_{BH}(t_i)\frac{m_{BH}(t)}{m_{BH}(t_i)}\left( \frac{t_i}{t} \right)^{2-n} \] (194)

The radiation density at the same time will be \( \rho_{rad}(t) = \rho(t) - \rho_{BH}(t) \).
Now we can have a formula for $\rho(t)$ solving the first Friedmann equation. Friedmann equations for $k = 0$ (flat universe) and including the BD - field $\phi$ are:

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} - \frac{\omega}{6} \phi^2 = \frac{8\pi \rho}{3\phi}$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2 \frac{\dot{a}}{a} \frac{\dot{\phi}}{\phi} + \frac{\omega}{2} \phi^2 + \frac{\dot{\phi}}{\phi} = -\frac{8\pi \rho}{\phi}$$

$$\frac{\ddot{\phi}}{8\pi} + 3 \frac{\dot{a}}{a} = \frac{\rho - 3p}{2\omega + 3}$$

(195)

Then we can use Eq. (193) for dust domination:

$$\frac{\dot{a}}{a} = \frac{2 - n}{3} t^{-1}$$

(196)

and also Eq. (190) for BH domination:

$$\phi = \frac{1}{G(t)} = \frac{1}{G_0} \left(\frac{t_{eq}}{t_0 t_{ev}}\right)^n t^{n} \Rightarrow \frac{\dot{\phi}}{\phi} = \frac{2 - n}{3} t^{-1}$$

(197)

Substituting these and also $\omega = \frac{2}{3n} - \frac{4}{3}$ to the first Friedmann equation, it becomes:

$$\rho(t) = \frac{n + 4}{8\pi G_0} \left(\frac{t_{eq}}{t_0 t_{ev}}\right)^n t^{n-2}$$

(198)

To calculate the baryon number produced by each one PBH, we have to know how their mass evolves with time due to accretion and evaporation.

$$\dot{m}_{acc} = 4\pi f R_{BH}^2 \rho_{rad}$$

(199)

where $f$ is accretion efficiency $\sim O(1)$. We set $f = 2/3$, as in [80]. $R_{BH} = 2Gm_{BH}$ is the radius of the BH.

$$\dot{m}_{ev} = -4\pi R_{BH}^2 a_H T_{BH}^4 = -\frac{a_H}{256\pi^3 G^2 m_{BH}^2} \left(\frac{t_{eq}}{t_0 t_{ev}}\right)^2 m_{BH}^2$$

(200)

where $a_H$ is the Stefan - Boltzmann constant. Combining accretion and evaporation and using $G(t)$ from Eq. (190) for BH - domination, we get:

$$\dot{m}_{BH} = 16\pi f G_0^2 \left(\frac{t_0 t_{ev}}{t_{eq}}\right)^{2n} t^{-2n} m_{BH}^2 \rho_{rad} - \frac{a_H}{256\pi^3 G_0^2} \left(\frac{t_0 t_{ev}}{t_{eq}}\right)^{-2n} t^{2n} m_{BH}^{-2}$$

(201)

At this point in order to analyze the whole scenario, we have to set some indicative values to our free parameters. Since we want to study a black hole dominated Universe from the moment of PBHs domination we select $\rho_{BH} = 2 \rho_{rad}$. 

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In order to have a feeling about the black hole masses that are relevant for our scenario we demand $\dot{m}_{BH} = 0$ and we find the initial BH mass for which accretion equals evaporation, $m_i \simeq 10^{25}\text{GeV}$ (or $\simeq 1\text{gr}$). Thus, we further choose $t_i = 10^{-30}\text{sec}$ and $m_i = 10^{27}\text{GeV}$. As shown in Fig. (3), accretion is able to increase the mass of the PBH, but only a little at the beginning. This is so because the radiation that was to be eaten becomes rapidly less dense, due to the universe expansion. The same, almost, happens for an initial mass even up to $10^{31}\text{GeV}$ (Fig. (4)). Only $10^{32}\text{GeV}$ or greater values accretion lead BHs to accumulate almost the entire universe mass (Fig. 5). We note here, though, that the value $m_{BH} = 10^{32}\text{GeV}$ is an upper limit, as shown in the bounds section.

Things are different in the case that PBH creation takes place earlier: $t_i \simeq 10^{-35}\text{sec}$. Initial accretion now equals evaporation for $m_i = 2.7 \times 10^{22}\text{GeV}$. Denser radiation makes accretion strong enough to lead to almost complete (95%) BH domination, for smaller initial masses (Fig. (6)). Then there is no more radiation for accretion to proceed.

The time that evaporation becomes stronger than accretion is given from Eq. (201), for $\dot{m}_{BH} = 0$ and for $m_{BH} = m_{\text{max}}$. For the $m_i = 10^{27}\text{GeV}$, $t_i = 10^{-35}\text{sec}$ case it is $m_{\text{max}} \sim 1.45 \times 10^{27}\text{GeV}$ and $t_{ev=ac} \sim 10^{-26}\text{sec}$. Universe will turn to radiation dominated with the evaporation of the PBHs. The evaporation and the result for the values in regard is shown in Fig. (7). The time of complete evaporation is for $m(t) = 0$ and it is $t_{ev} \simeq 2.7 \times 10^{-17}\text{sec}$.

### 6.3.1 Baryogenesis

In the following, we calculate the baryon number generated by a single black hole and then the baryon to entropy ratio $b/s$ of the universe.

Although sphaleron process takes place both in the symmetric region around a black hole and the domain wall, the required CP - violation and non-equilibrium conditions coexist only in the domain wall. So, it is there that the baryon assymetry is created. In addition, $f(r) = |\langle \phi_2(r) \rangle|/v \leq \epsilon = 1/100$ is needed, so as the order of the sphaleron process exponential factor to be one and the baryon asymmetry not to be suppressed. In other words, baryon generation happens in the region of the domain wall that Higgs scalar value is small and this is from $r_{DW}$ to $r_{DW}+d_{sph}$, where $d_{sph}$ is defined from $f(r_{DW}+d_{sph}) = \epsilon$. Then, it is $\int_{r_{DW}+d_{sph}}^{r_{DW}} dr \frac{d}{dr}\varphi(r) = \epsilon \Delta \varphi_{CP}$, where $\varphi(r, T) = [f(r) - 1]\Delta \varphi_{CP}$ [43]. Thus,
\[
\dot{B} = V \frac{\Gamma_{sph}}{T_W} \mathcal{N} \dot{\phi} \\
= 4\pi \mathcal{N} \kappa \alpha_W^5 T_W^3 r_{\text{DW}}^2 v_{\text{DW}} \int_{r_{\text{DW}}}^{r_{\text{TDW}}+d_{\text{ph}}} \frac{d}{dr} \frac{\phi(r)}{r} \\
= \frac{1}{16\pi} \mathcal{N} \kappa \alpha_W^5 \epsilon \Delta \varphi_{\text{CP}} \frac{T_{\text{BH}}^2}{T_W}
\]

(202)

where \(\Gamma_{sph}\) is the sphaleron transition rate, \(\Delta \varphi_{\text{CP}}\) the net CP phase. \(\mathcal{N} \simeq O(1)\) is a model dependent constant which is determined by the type of spontaneous electroweak baryogenesis scenario and the fermion content, \(\kappa \simeq O(30)\) is a numerical constant expressing the strength of the sphaleron process. This is the same as in 4-d GR.

Integrating numerically through the BHs lifetime, we calculate the total baryon number by a single BH.

\[
B = \int_{t_i}^{t_{\text{ev}}} \dot{B} \, dt
\]

(203)

The baryon number produced during accretion is orders of magnitude smaller than during evaporation.

After the BHs have gained their maximum mass, they only evaporate at a very slow rate until the very last moments before their complete annihilation (Fig. (7)). Thus, we can use an approximation where BHs mass remains constant until the time of evaporation when it turns to radiation completely.

The total baryon number density produced is:

\[
b = B \frac{\rho_{\text{BH}}(t_{\text{ev}}^-)}{m_{\text{max}}}
\]

(204)

where

\[
\rho_{\text{BH}}(t_{\text{ev}}^-) = \rho_{\text{rad}}(t_{\text{ev}}^+) = \frac{\pi^2}{30} g_{\text{reh}} \frac{T_{\text{reh}}^4}{T_{\text{reh}}^4}.
\]

(205)

\(T_{\text{reh}}\) is the temperature that the universe is reheated as BHs evaporate. We set \(T_{\text{reh}} = 95\text{GeV}\) so as to be below \(T_W\). \(m_{\text{max}}\) is 1.5\(m_i\), as shown before. The entropy density is

\[
s = \frac{2\pi^2}{45} g_{\text{reh}} T_{\text{reh}}^3
\]

(206)

where \(g_{\text{reh}}\) is the massless degrees of freedom of the reheated plasma in the asymmetric phase.
Finally \( b/s \sim 8.7 \times 10^{-10} \Delta \theta_{CP} \). In order to have the observed \( 6 \times 10^{-10} \) we need \( \Delta \theta_{CP} \sim 0.7 \text{rad} \). Such a CP-violation phase is very possible for a 2HDM [82].

### 6.4 Second case: Primordial black holes domination because of accretion

Another case, even more interesting, is the one where the PBHs, at the time of their creation, are only a small fraction of the total universe density and the universe is radiation dominated. As it will be shown, accretion can be strong enough to lead to PBH domination and the production of the observed baryon number.

In this scenario, PBHs form also around the end of the \( \phi \)-domination era, but they consist of only a portion of \( \rho \). That means a radiation domination period begins after the \( \phi \)-domination era. If accretion is able to lead most radiation inside the BHs, a PBH domination epoch follows, after \( t_{eq1} \). \( t_{eq1} \) is the time BHs density becomes equal to radiation density. The universe turns radiation dominated for the second time with BHs evaporation.

The evolution of \( G(t) \) now is (if accretion lead from radiation to PBH domination):

\[
G(t) \simeq G_0 \left( \frac{t_i}{t} \right)^{(n+1) \left( \frac{t_0 t_{ev}}{t_{eq} t_{eq1}} \right)^n}, \text{ if } t < t_i : \text{BD – field dominated}
\]
\[
G_0 \left( \frac{t_0 t_{ev}}{t_{eq} t_{eq1}} \right)^n, \text{ if } t_i < t < t_{eq1} : \text{radiation dominated}
\]
\[
G_0 \left( \frac{t_0 t_{ev}}{t_{eq1} t_{eq}} \right)^n, \text{ if } t_{eq1} < t < t_{ev} : \text{PBHs dominated}
\]
\[
G_0 \left( \frac{t_{eq1}}{t_{eq}} \right)^n, \text{ if } t_{ev} < t < t_{eq} : \text{radiation dominated}
\]
\[
G_0 \left( \frac{t_0}{t} \right)^n, \text{ if } t_{eq} < t : \text{dust dominated}
\]

(207)

For the period \( t_i < t < t_{eq1} \), \( G = \text{constant} \), as one can see at Eq. (207) for radiation domination, and so \( \dot{\phi} = 0 \). Then, the first Friedmann equation (Eq. (195)) becomes:

\[
\frac{a^2}{a^2} = \frac{8\pi \rho}{3\phi}
\]

(208)

where \( a \propto t^{1/2} \) and thus we have the total density \( \rho(t) \). Eq. (192) for the PBHs energy density holds. Substituting the corresponding \( a \):
\[
\rho_{BH}(t) = \rho_{BH}(t_i) \frac{m_{BH}(t)}{m_{BH}(t_i)} \left( \frac{t_i}{t} \right)^{3/2}, \quad t_i < t < t_{eq1}
\]
\[
= \rho_{BH}(t_{eq1}) \frac{m_{BH}(t)}{m_{BH}(t_{eq1})} \left( \frac{t_{eq1}}{t} \right)^{2-n}, \quad t_{eq1} < t < t_{ev}
\]

For radiation it is still \( \rho_{rad}(t) = \rho(t) - \rho_{BH}(t) \).

The mass evolution of the PBHs is determined, as in the previous case, by accretion (Eq. (199)) and evaporation (Eq. (200)). The limit for accretion to be stronger than evaporation is now \( m_{BH}(t_i) \sim 2 \times 10^{22} \text{GeV} \). In Fig. (8), (9) is shown the mass evolution during the accretion period. One can see that accretion is very effective for \( m_i \geq 10^{27} \text{GeV} \) and as a consequence the universe becomes almost completely PBH dominated. With 95% of the density inside the BHs, there is nothing else to accrete. Evaporation follows (Fig. (10)).

So, the mass of a single PBH can increase up to 100,000 times (from \( 10^{27} \) to \( 10^{32} \) GeV) because of accretion. The black hole lifetime also increases. The mechanism of baryon number production is the same as in the previous case and thus the baryonic asymmetry created by a single PBH is considerably enhanced. The total baryon number to entropy density is calculated \( b/s \sim 8.7 \times 10^{-10} \Delta \theta_{CP} \), compatible with the observed one. This is the same with the previous case because the baryon number density \( b \) is proportional to the PBHs density, which in both cases is almost the total density of the universe.

### 6.5 Bounds

One limit for PBHs mass is posed by the fact that the size of the domain wall \( d_{DW} \) must be greater than the mean free path (MFP) \( \lambda \).

\[
d_{DW} = \frac{9}{256\pi^2} \frac{1}{\beta_{SM} \Gamma W} \frac{T_{BH}^2}{T_{W}^3}
\]

\[
\lambda = \frac{\beta}{T_{W}}
\]

\[d_{DW} > \lambda \Rightarrow T_{BH} > 53 \text{TeV} \Rightarrow m_{BH} < 10^{32} \text{GeV}\.

Another limit appears because the black hole lifetime \( \tau_{BH} \) should be quite greater than the time for the stable weak domain wall to form. Eq. (200) is integrated analytically:
\[ m(t) = \frac{3^{1/3}((f - 1)a_H t_0^{-2n} t_{eq}^{-2n} t_1^{-1+2n} + 256 G_0^2 \pi^3 (1 + 2n)m_{\text{max}}/3)^{1/3}}{4 \times 2^{2/3} \Gamma_0^{2/3} (1 + 2n)^{1/3} \pi} \] (213)

We use the formula for BH lifetime without accretion (that is from \( m_{\text{max}} \) till complete evaporation) because the evaporation period of BH life is orders of magnitude greater than the accretion period. We take the evaporation period as the time duration from the moment that the evaporation becomes stronger than accretion, until the moment of PBH complete annihilation.

\[ \tau_{BH} \sim \frac{256 G_0^2 m_{\text{max}}^3 (1 + 2n) \pi^3 t_0^{-2n} t_{eq}^{-2n}}{3 a_H(1 - f)} . \] (214)

The domain wall formation time is

\[ \tau_{DW} = \frac{d_{DW}}{u_{DW}} = \frac{27 T_{BH}^4}{4096 \pi^4 \beta_{SM}^3 c_W T_W^5} \] (215)

Solving for \( m_{\text{max}} \) we find that it should be, approximately \( m_{\text{max}} > 10^{28} \text{GeV} \). The masses that provide successful baryogenesis in our model are within these limits. To avoid confusion, this second constraint provides a lower bound on masses. The parameter \( m_{\text{max}} \) refers to the maximum value after accretion finishes to be dominant.

### 6.6 Primordial black holes mass spectrum

In the previous sections we worked with the assumption that all the black holes have the same mass. Thus, it was possible to have some analytical solutions, to check if the model produces the observed baryon number and to set bounds on black holes’ mass. Yet, it is more natural to assume that there is a spectrum of the initial masses. So, we are going to examine how this affects our model.

The two limits set in the previous section are still valid in the case of mass spectrum, since they refer to each one black hole’s mass. PBHs with mass greater than the upper bound are not hot enough to thermalize their neighbourhood. If they have, on the other hand, mass less than the lower bound, then their lifetime is not long enough to form the domain wall where the baryogenesis would take place. Only the part of PBHs mass spectrum in the range between the two bounds contributes to the baryon number generation.
Eqs. (202), (203) for the baryon number created by a single PBH are still valid, but the total baryon asymmetry created by all PBHs is

\[ b = \int_{0}^{\infty} B N(m,t) \, dm, \]  

(216)

where \( N \) is the number density of PBHs with masses from \( m \) to \( m + dm \). As a general conclusion, it suffices to state that the very efficient baryogenesis due to accretion remains unaffected from the presence of mass spectrum. Based on a certain cosmological scenario of the creation of PBHs one can estimate the exact baryon asymmetry straightforwardly. More details will follow concerning the relation of the black hole mass spectrum and the time evolution of the scale factor and the cosmic densities.

Next, we derive the equations governing the evolution of the spectrum of PBHs. We assume that the initial number density of the black hole spectrum is described by a power-law form, as in [51] and [85]. Thus, the initial number density of the PBHs with masses between \( m_0 \) and \( m_0 + dm_0 \) is

\[ N(m_0) dm_0 = A m_0^n \Theta(m_0 - m_c) dm_0, \]  

(217)

where \( m_0 = m(t = 0) \) is the initial PBH mass. For the analytic calculations not to become unnecessarily complicated, we accept that all PBHs form at the same initial time \( t_0 \). We use \( \Theta \) to introduce a cut-off mass \( m_c \). This protects from divergences at the low-masses limit. Thus, we set \( \Theta = 1 \) for \( m > m_c \) and \( \Theta = 0 \) for \( m \leq m_c \). We assume that \( m_c \) is proportional to the Planck mass, \( m_c = k m_{pl} \), where the constant \( k \) is arbitrary and has no dimensions. For the total energy density not to diverge at large masses, it has to be \( n > 2 \). According to Carr [51], initial density perturbations that produce PBHs in standard cosmology, indicate that \( n \) is between 2 and 3. \( A \) is the amplitude of the spectrum. Its units are such that \( N(m_0) dm_0 \) is number density.

The total number density of the black holes, as a function of time, is

\[ N(t) = \int_{0}^{\infty} N(m,t) \, dm, \]  

(218)

and their total energy

\[ \varrho_{BH}(t) = \int_{0}^{\infty} N(m,t) \, m \, dm. \]  

(219)

Treating analytically the evolution of the mass spectrum considering both accretion and evaporation, was not possible. Yet, in our model the epoch when accretion is dominant is succeeded very quickly by an epoch when evaporation prevails, resulting in a reheated, radiation dominated universe. Thus, one can treat the two epochs separately.
6.6.1 Dominant accretion time period

Here we will analyze the time period after primordial black hole creation where the accretion is important. Our aim is to calculate the evolution of black holes and radiation densities and the scale factor.

The factors that determine the PBHs mass spectrum evolution are not only the universe’s expansion, but accretion also. The rate of gain, because of accretion, for a single black hole is given by Eq.(199). Solving it we get

\[ m_0 = \frac{1}{m(t)^{-1} + 16 \pi f I_\rho}, \]  

(220)

where \( I_\rho = \int_0^t G^2 \rho_{\text{rad}}. \)

Differentiating Eq.(220) with respect to \( m_0 \), we can have an expression for the evolution of the number density of PBHs with masses from \( m \) to \( m + dm \) at time \( t \), combining it with Eq. (217) (special care must be given for the jacobian factor). So, the evolution of the mass spectrum with time is

\[ N(m,t)dm = N(m_0,t)dm_0 = A \left( \frac{1}{m(t)^{-1} + 16 \pi f I_\rho} \right)^{2-n} m^{-2} \Theta(m - m_{ca}(t)) dm, \]  

(221)

where \( m_{ca} \) is the cut-off mass that is evolved from \( m_c \):

\[ m_{ca}(t) = \left( \frac{1}{k m_{pl} - 16 \pi f I_\rho} \right)^{-1}. \]  

(222)

One can see that, contrary to the evaporation epoch, the cut-off mass never becomes 0.

The energy density rate is determined using the identity

\[ \frac{d}{dx} \int_{g(x)}^{f(x)} h(x,y) dy = \int_{g(x)}^{f(x)} \frac{\partial h(x,y)}{\partial x} dy + h(x,f(x)) \frac{df(x)}{dx} - h(x,g(x)) \frac{dg(x)}{dx}. \]  

(223)

The energy density of the radiation that is eaten by the PBHs and so is added to the black hole density \( \varrho_{bh} \) (not the comoving), from time \( t \) to \( t + dt \), is

\[ dE = \varrho_{bh}(t + dt) - \varrho_{bh}(t) = \frac{\partial \varrho_{bh}}{\partial t} dt. \]

The energy density rate, then, is calculated using also Eq. (223):

\[ \frac{dE}{dt} = \frac{d}{dt} \int_0^\infty N(m,t) m dm = \int_0^\infty A(n-2) (m^{-1} + \xi I_\rho)^{n-3} \xi \frac{dI_\rho}{dt} m^{-1} dm - A (m_{ca}^{-1} + \xi I_\rho)^{n-2} m_{ca}^{-1} \frac{dm_{ca}}{dt} \Theta(m - m_{ca}(t)). \]  

(224)

where \( \xi = 16 \pi f \) and

\[ \frac{dm_{ca}}{dt} = \left( \frac{1}{k m_{pl}} - \xi I_\rho \right)^{-2} \frac{dI_\rho}{dt}. \]  

(225)
The first term of Eq. (224) is actually the evolution of the spectrum. The second term is present because of the mass cut-off evolution. In the accretion era this term does not vanish, since evaporation is insignificant, compared to accretion. Then, we can have the full equations that determine the expansion, where the densities must be multiplied by \( a^{-3} \), in order to become comoving.

The resulting set of equations is

\[
\rho_{BH} = \frac{1}{a^3} \int_0^\infty N(m, t) m \, dm, \quad \rho = \rho_{BH} + \rho_{rad} \tag{226}
\]

\[
\frac{dE_{com}}{dt} = \frac{1}{a^3} \frac{dE}{dt} \tag{227}
\]

\[
\dot{\rho}_{rad} = -\frac{4}{a} \rho_{rad} - |\frac{dE_{com}}{dt}|, \tag{228}
\]

since the kinetic pressure by the black holes is not important.

This set is supplemented by Eqs. (195) and either (190) for the first case or (207) for the second case. They are an integro-differential system, which is solved only numerically for various ranges of the parameters.

### 6.6.2 Dominant evaporation time period

At some point in time accretion becomes less important than evaporation. This happens due to the ongoing expansion of the universe and, mainly, because the whole of the universe’s radiation ends inside the PBHs, as we explained in the previous sections. From that time on, evaporation dominates the evolution of the black hole mass. The significance of this analysis lies in finding the modifications to the expansion rate, allowing the emergence of the conventional radiation expansion law. We aim to determine the deviations of the PBHs and of radiation densities and the evolution of the scale factor.

The evolution of the PBHs mass spectrum depends on the expansion of the universe and, more importantly, on the evaporation of the PBHs. The rate of mass loss of a single black hole, because of evaporation, is given by Eq.(200). Integrating it we get Eq.(213). Note that now the initial value \( m_0 = m_{\text{max}} \) is the maximum value of the black hole mass after the end of the dominant accretion time period.

\[
m^3 = m_0^3 - \frac{3 a_H}{256 \pi^3} I_g \tag{229}
\]

where \( I_g = \int_0^t G^{-2} dt \). Then, we solve with respect to \( m_0 \) and differentiate. Thus, we can have the evolution of the black holes number density from \( m \) to \( m + dm \) at time \( t \) from
Eq.(217). The evolved mass spectrum is given by

\[ N(m, t) \, dm = A \left( m^3 + \frac{3a_H}{256\pi^3} I_g \right)^{-\frac{(n+2)}{3}} m^2 \Theta(m - m_{cr}(t)) \, dm, \]  

where the cut-off mass is evolved, too:

\[ m_{cr}(t) = [(k m_{pl})^3 - \frac{3a_H}{256\pi^3} I_g]^{1/3}. \]  

We can see that there is a time \( t_{lim} \) that the cut-off mass becomes 0.

The energy density that is emitted by the black hole as radiation from \( t \) to \( t + dt \) is estimated from Eq. (219). It is

\[ dE = \varrho_{bh}(t) - \varrho_{bh}(t + dt) = -\frac{\partial \varrho_{bh}}{\partial t} \, dt. \]  

where the quantities are not comoving. We can have the energy density rate using the identity Eq.(223). So, we find

\[ \frac{-dE}{dt} = \frac{d}{dt} \int_{0}^{\infty} N(m, t) \, m \, dm \]  

\[ = A \frac{a_H}{256\pi^3} \left( -n - 2 \right) \int_{m_{c,max}}^{\infty} m^3 \left( m^3 + \frac{3a_H}{256\pi^3} I_g \right)^{-\frac{(n+5)}{3}} \frac{dI_g}{dt} \, dm \]  

\[ -A m_{cr}^3 \left( m_{cr}^3 + \frac{3a_H}{256\pi^3} I_g \right)^{-\frac{(n+2)}{3}} \Theta(m - m_{cr}(t)) \frac{d m_{cr}}{dt}, \]  

where

\[ \frac{d m_{cr}}{dt} = -\frac{a_H}{256\pi^3} \left[(k m_{pl})^3 - \frac{3a_H}{256\pi^3} I_g\right]^{-2/3} \frac{dI_g}{dt} \]  

and

\[ m_{c,max}(t) = \max[0, m_{cr}(t)]. \]

The first term in Eq. (234) expresses the evolution of the spectrum and is the only non-zero term at late times. The second term of Eq. (234) is present because of the time evolution of the mass cut-off. It is apparent that for times larger than \( t_{lim} \) the lightest black holes completely evaporate and the \( \Theta \) function causes this term to vanish.

In all the quantities calculated so far, the dilution from the expansion will have to be added; the comoving density is \( \rho_{BH} = \varrho_{bh} a^{-3} \), and the comoving energy is \( E_{com} = Ea^{-3} \).

Finally, the set of equations is the following

\[ \rho_{BH} = \frac{1}{a^3} \int_{0}^{\infty} N(m, t) \, m \, dm, \quad \rho = \rho_{BH} + \rho_{rad} \]  

71
\[
\frac{dE_{\text{com}}}{dt} = \frac{1}{a^3} \frac{dE}{dt}, \quad (238)
\]

\[
\dot{\rho}_{\text{rad}} = -4 \frac{\dot{a}}{a} \rho_{\text{rad}} + |\frac{dE_{\text{com}}}{dt}|, \quad (239)
\]

since black holes exert unimportant kinetic pressure.

Eqs. (237), (238), (239), (195) and either (190) for the first case or (207) for the second case are an integro-differential system. Like in the dominant accretion time period, the equations system can be solved only numerically for various ranges of the parameters.

6.7 Conclusions

Primordial black holes born in a BD-universe, at the end of the BD-field domination era (\(\sim 10^{-35} \text{ sec}\)), with initial mass \(m_i \geq 10^{27}\text{ GeV}\), accrete radiation from their surroundings intensively, leading to almost complete PBH domination, even if PBHs density was initially only 1/100,000 of the universe density. However, the maximum of PBH mass should not exceed \(\sim 10^{32}\text{ GeV}\).

The produced baryon to entropy is \(b/s \sim 8.7 \times 10^{-10} \Delta \theta_{\text{CP}}\) and can match the observed \(6 \times 10^{-10}\) value with a loop corrected \(\Delta \theta_{\text{CP}} \sim 0.7\text{ rad}\), which is within the limits of phenomenologically accepted two Higgs doublet models.

We proved that BD gravity, due to enhanced accretion, can naturally provide black holes domination in the early Universe and at the same time, efficient baryogenesis for smaller CP violating angles compared to the case of the conventional gravity of General Relativity.

It is worth studying the ideas presented in this work for Asymptotic Safe Gravity [87], since it shares some similar properties to Brans-Dicke models. Another interesting question is to analyse how initial anisotropic or inhomogeneous backgrounds (with small anisotropies/inhomogeneities that smooth out later) affect the mechanism [88].
Figure 3: $t_i = 10^{-30} \text{sec}$: initial $\rho_{BH} \sim 67\% \rho_{universe}, \rho_{rad} \sim 33\% \rho_{un}, M_i = 10^{27} \text{GeV}$: accretion is insufficient to increase BH mass significantly.

Figure 4: $t_i = 10^{-30} \text{sec}$: initial $\rho_{BH} \sim 67\% \rho_{un}, \rho_{rad} \sim 33\% \rho_{un}, M_i = 10^{31} \text{GeV}$: accretion is insufficient to increase BH mass significantly.
Figure 5: for $t_i = 10^{-30}$ sec, $M_i = 10^{32}$ GeV: $\rho_{\text{rad}} \sim 33\% \rho_{\text{un}}$, for $t \sim 10^{-27}$ sec, $M_{BH} \sim 1.45 \times 10^{32}$ GeV: $\rho_{\text{rad}} \sim 3\% \rho_{\text{un}}$: accretion is sufficient to lead to almost total BH domination.

Figure 6: for $t_i = 10^{-35}$ sec, $M_i = 10^{27}$ GeV: $\rho_{\text{rad}} \sim 33\% \rho_{\text{un}}$, for $t \sim 10^{-32}$ sec, $M_{BH} \sim 1.45 \times 10^{27}$ GeV: $\rho_{\text{rad}} \sim 3\% \rho_{\text{un}}$: accretion is sufficient to lead to almost total BH domination.
Figure 7: $\rho_{BH} = 2 \rho_{rad}, M_i = 10^{27} GeV, t_i = 10^{-35} sec, evaporation$

Figure 8: $t_i = 10^{-35} sec: initial \rho_{BH} \sim 10^{-6} \rho_{un}, \rho_{rad} \sim \rho_{un}, M_i = 10^{26} GeV: accretion is insufficient to increase BH mass significantly$
Figure 9: For $t_i = 10^{-35}\,\text{sec}$: $M_i = 10^{27}\,\text{GeV}$, $\rho_{BH} \sim 10^{-3}\,\rho_{un}$, $\rho_{rad} \sim \rho_{un}$. For $t \sim 10^{-32}\,\text{sec}$: $M_{BH} \sim 10^{30}\,\text{GeV}$, $\rho_{BH} \sim \rho_{un}$, $\rho_{rad} \sim 0$: accretion is sufficient to lead to almost total BH domination.

Figure 10: $t_i = 10^{-35}\,\text{sec}$, $\rho_{BH} = 10^{-3}\,\rho_{rad}$, $M_i = 10^{27}\,\text{GeV}$, $M_{BH(max)} \sim 10^{30}\,\text{GeV}$, evaporation.
7 Neutrino mass and mixing

7.1 Neutrino mass

Neutrinos are very light particles. We don’t know, however, exactly how light. Even today the exact values of their masses are unknown. There are only upper limits. Moreover, the standard model does not predict neutrinos to have mass. For a neutrino masses review see [89].

Charged leptons mass terms are created by coupling Higgs scalar doublet $\phi$ with left-handed lepton doublets $L_L$, which consist of a charged lepton, eg. electron, and the associated neutrino, and right-handed singlets $L_R$:

$$L_{Yukawa,lep} = -\frac{\sqrt{2}}{v} L_L m_{lep} L_R \phi. \quad (240)$$

Right-handed neutrinos, however, have not been observed and are not included in the Standard Model. If they exist, they are SU(2) singlets and so they do not interact even via the weak force. They would interact only gravitationally and so, their observation would be very difficult. This is why they are called sterile neutrinos.

When the Standard Model is extended with $n$ sterile neutrinos (it is not obligatory for right-handed neutrinos to be equal in number with the left-handed), two different types of mass terms can be written:

$$-L_{M_\nu} = M_{D\nu} \bar{\nu}_s \nu_L \phi + \frac{1}{2} M_{N\nu} \bar{\nu}_s \nu^c_s, \quad (241)$$

where $\nu^c$ is the conjugated neutrino, matrix $M_D$ has dimension $n \times 3$ and symmetric matrix $M_N$ $n \times n$. The first term is the Dirac mass term, produced by spontaneous electroweak symmetry breaking from Yukawa interactions, like all the other masses in the Standard Model. The second term is a Majorana mass term. Such a term is allowed only for particles that do not carry any additive conserved charge, like the electric charge. This is so because of the appearance of the charge conjugate in the term.

Eq. (241) can be also written as

$$-L_{M_\nu} = \frac{1}{2} \left( \begin{array}{c} \bar{\nu}_L^c, \bar{\nu}_s \end{array} \right) \left( \begin{array}{cc} 0 & M_D^T \\ M_D & M_N \end{array} \right) \left( \begin{array}{c} \bar{\nu}_L^c \\ \bar{\nu}_s^c \end{array} \right), \quad (242)$$

where

$$M_\nu = \left( \begin{array}{cc} 0 & M_D^T \\ M_D & M_N \end{array} \right) \quad (243)$$

is the neutrino mass matrix.
7.1.1 See-saw mechanism

The eigenvalues of the mass matrix $M_\nu$ are either proportional to $M_N$ or to $M_N^{-1}$. If $M_N$ is very high, then $M_N^{-1}$ is very low and this is why the mechanism is named see-saw. Thus, the extremely low masses ($<1\text{eV}$) of the known neutrinos are explained by assuming the existence of very heavy sterile neutrinos with masses at energy scale much higher than that of the electro-weak symmetry breaking, perhaps at the scale of the Grand Unification ($\sim 10^{15}\text{GeV}$).

We remind here that, according to the Baryo-through-Leptogenesis models, the heavy neutrinos predicted by the See-saw mechanism can also create lepton number with their decay, when the universe’s temperature falls below their mass scale. The excess of leptons over anti-leptons is then transformed to an excess of baryons over anti-baryons via sphaleron processes. Heavy sterile neutrinos are candidates for the WIMPS (weakly interacting massive particles) that may constitute the dark matter.

7.2 Neutrino oscillations

Another problem with neutrinos that was unsolved for over three decades was the famous Solar Neutrinos Problem. In the late 1960s, J. N. Bahcall calculated the flux of neutrinos produced by the sun. A neutrino observatory was built to detect the flux of solar neutrinos. That was the Homestake experiment, led by R. Davis. It was measured only one-third of the predicted flux. The problem remained unsolved until the end of the 20th century.

The solution to one problem is also a solution to the other: Neutrinos do have mass, even though of very low value, less than 1eV. Neutrinos flavor eigenstates are the three types of neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$. There are also three mass eigenstates $\nu_1$, $\nu_2$, $\nu_3$, but they are not the same as the flavor eigenstates. Neutrinos propagate as mass eigenstates, not flavor ones. So there is a non zero probability for a neutrino to change its flavor when propagating. This is "neutrino oscillation" or "neutrino mixing".

The theory of neutrinos oscillations was formulated by Z. Maki, M. Nakagawa and S. Sakata in 1962 [90]. It was further developed by B. Pontecorvo in 1968 [91]. If one accepts that neutrinos have mass and also that the mass eigenstates are different than the flavour eigenstates, then the mass eigenvector transforms to the flavour eigenvector when multiplied by a unitary matrix $U$:

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= 
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}.
$$

(244)
This transformation can be regarded as a product of three Euler rotations (figure 11). Consequently U can be parametrized as:

\[
U = \begin{pmatrix}
 c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-ia} \\
 -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{ia} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{ia} & c_{13}s_{23} \\
 -c_{12}c_{23}s_{13}e^{-ia} + s_{12}s_{23} & -c_{23}s_{12}s_{13}e^{-ia} - c_{12}s_{23} & c_{13}c_{23}
\end{pmatrix},
\]

where \( c_{ij} \equiv \cos \theta_{ij} \) and \( s_{ij} \equiv \sin \theta_{ij} \). \( \theta_{ij} \), \( (\theta_{12}, \theta_{23}, \theta_{13}) \) are the mixing angles and \( a \) is the CP violating angle. This mixing matrix is also called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. In case neutrinos are Majorana particles, that is they are their own anti-particles, two more phases, the Majorana phases \( n_1 \) and \( n_2 \) are involved. Then, the mixing matrix is the one in eq. (276) multiplied by

\[
P = \begin{pmatrix}
 e^{in_1} & 0 & 0 \\
 0 & e^{in_2} & 0 \\
 0 & 0 & 1
\end{pmatrix}.
\]

The experimental ranges of values of the angles can be found in the Review of Particle Physics (RPP) [92]. The 3\( \sigma \) range is

\[
\begin{align*}
\sin^2 \theta_{12} &= [0.25 - 0.35] \\
\sin^2 \theta_{23} &= [0.38 - 0.62] \\
\sin^2 \theta_{13} &= [0.0185 - 0.0246].
\end{align*}
\]
or, approximately

\[ \sin^2 \theta_{12} \approx 0.31, \sin^2 \theta_{23} \approx 0.55, \sin^2 \theta_{13} \approx 0.022 \]  \hspace{1cm} (248)

The first two values, \( \theta_{12} \) and \( \theta_{23} \), are quite close to \( 1/3 \) and \( 1/2 \) respectively. The value of \( \theta_{13} \) is small and in the past, it was considered that it could be even 0. Later, as one can see in eq. (247), this was excluded. It could be \( \theta_{13} = 0 \) only in the zeroth order approximation, corrected then by higher order terms. The tri-bimaximal is the mixing matrix with these values [93]:

\[
U_{\text{TB}} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]  \hspace{1cm} (249)

7.3 Neutrino mixing and discrete symmetry

As shown in the previous section, the experimental values of the neutrino mixing matrix elements do not seem to be arbitrary. This fact indicates the existence of an underlying non-Abelian discrete flavor symmetry. Non-abelian and abelian discrete symmetries have been used to impose constraints on the Yukawa lagrangian. They have been used to produce hierarchical structures in the fermion mass matrices, to suppress proton decay operators (since the proton decay has not been observed) or other terms inducing unobserved processes. In this frame, the neutrino mass matrix is assumed to be invariant under certain transformations of the discrete group \( D_f \) ([94, 95, 96, 97]). Therefore, in unified theories of the fundamental gauge interactions, the symmetry of the effective model is expected to contain a non-abelian gauge group \( G_{\text{GUT}} \) accompanied by a (non)-abelian discrete flavor symmetry \( D_f \). In some string models the total effective symmetry \( G_{\text{GUT}} \times D_f \) is usually embedded in a higher unified group, such as \( E_8 \) [98]. For GUT symmetries such as \( E_6, SO(10) \) and \( SU(5) \), the discrete group is a subgroup of \( SU(3) \). Several of these cases have been considered, including those belonging to \( S_n, A_n \) and possess triplet representations.

The pattern observed in the mixing matrix can be reproduced by some flavor discrete symmetry corresponding to a non-Abelian finite group. This symmetry could be valid at high energies and broken at low energies, leaving subgroups for the charged leptons and the neutrinos. Such groups that have been used to produce the mixing matrix of neutrinos
Table 1: Some finite groups used to reproduce neutrino mixing matrix.

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of elements</th>
<th>Generators</th>
<th>Irreducible representations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_4$</td>
<td>24</td>
<td>$S, T(U)$</td>
<td>$1, 1', 2, 3, 3'$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>12</td>
<td>$S, T$</td>
<td>$1, 1', 1'', 3$</td>
</tr>
<tr>
<td>$T'$</td>
<td>24</td>
<td>$S, T(R)$</td>
<td>$1, 1', 1'', 2, 2', 2'', 3$</td>
</tr>
<tr>
<td>$A_5$</td>
<td>60</td>
<td>$\tilde{S}, \tilde{T}$</td>
<td>$1, 3, 3', 4, 5$</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>20</td>
<td>$A, B$</td>
<td>$1_1, 1_2, 1_3, 1_4, 2_1, 2_2, 2_3, 2_4$</td>
</tr>
<tr>
<td>$D_{12}$</td>
<td>24</td>
<td>$\tilde{A}, \tilde{B}$</td>
<td>$1_1, 1_2, 1_3, 1_4, 2_1, 2_2, 2_3, 2_4, 2_5$</td>
</tr>
</tbody>
</table>

are $S_4$, $A_4$, $A_5$, $\Delta$, $D_{10}$ and $D_{12}$, $T'$ (Table 1). A common characteristic of them is that they describe symmetries with respect to large angle rotations. This is useful because angles $\theta_{12}$ and $\theta_{23}$ of the mixing matrix are quite large. The $\theta_{13}$ they produce is zero or it is small. Perturbative corrections to meet the experimental ranges of angle values can be provided by the charged leptons mass diagonalizing matrix $V_e$. The mixing matrix is

$$U = V_e^\dagger V_n,$$  \hspace{1cm} (250)

where $V_n$ is the neutrino mass diagonalization matrix.

Some characteristics of models based on discrete symmetries, as expressed in [94] are:

1. In several models it is $\sin^2 \theta_{23} = 0.5$ exactly.

2. In some models angles $\theta_{23}$ and $\theta_{13}$ are correlated via

$$\sin^2 \theta_{23} = 0.5(1 + \sin^2 \theta_{13} + O(\sin^4 \theta_{13}))$$  \hspace{1cm} (251)

3. $\sin^2 \theta_{23}$ is different from 0.5 with very small uncertainty, in some models: $\sin^2 \theta_{23} = 0.455, 0.463, 0.537, 0.545$.

4. $\theta_{12}$ and $\theta_{13}$ are related, in some models, as:

$$\sin^2 \theta_{12} = 1/(3 \cos^2 \theta_{13}) = (1 + \sin^2 \theta_{13} + O(\sin^4 \theta_{13}))/3 \cong 0.340.$$  \hspace{1cm} (252)

5. In some other models, though, their correlation is different:

$$\sin^2 \theta_{12} = (1 - 3 \sin^2 \theta_{13})/(3 \cos^2 \theta_{13}) = (1 - 2 \sin^2 \theta_{13} + O(\sin^4 \theta_{13}))/3 \cong 0.319.$$  \hspace{1cm} (253)
6. In a large class of models, called “one-parameter models”, the mixing matrix is a function of just one real continuous free parameter. Then, the Dirac and Majorana phases are CP-conserving, with values 0 or $\pi$. Only in few such models it is $\delta = \pi/2$ or $3\pi/2$.

7. In those models that the mixing matrix elements are functions of two angles and (in some of them) a phase, the Dirac phase $\delta$ can be expressed in terms of the mixing angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ and one or more fixed (known) parameters $\theta^\nu$ which depend on the discrete symmetry of the model.
8 Neutrino mixing matrix generated by $PSL(2, 7)$ elements

The present chapter is based on [3]. Here we investigate whether any elements of the $PSL(2, 7)$ finite group can be used to generate a neutrino mixing matrix that respects the known observational and experimental bounds (Eq. 247).

A discrete symmetry may be underlying the structure of the neutrinos mixing matrix, as mentioned in the previous chapter. Finite groups exhibit such symmetries and for this reason, they have been used in many models to explain the characteristic form of the neutrino mixing matrix. Here we assume that the neutrino and the charged lepton mixing matrices commute with elements of the same or different subgroups of a parent discrete symmetry. Such constraints that are forced on the mass matrices have been used in model building in the framework of some string theory. In some F-theory framework for example we assume the $SU(5)$ gauge theory and then the trilinear Yukawa couplings for the various types of fields are realized at different points of the internal manifold and they correspond to different symmetry enhancements of the $SU(5)$ singularity [105].

A general class of discrete symmetries is that of the special linear groups $SL(2, p)$ [99, 100] and the relative projective ones, $PSL(2, p)$, where $p$ is a prime number. For $p \leq 7$ the groups are subgroups of $SU(3)$ and also they have three-dimensional representations. $PSL(2, 7)$ is not much explored (although [101, 102, 103]) and it is a simple subgroup of $SU(3)$.

In order to use the $PSL(2, 7)$ finite group, we assume that the neutrino mass matrix commutes with an element $A$ of the $PSL(2, 7)$ and that the same happens for the charged lepton mixing matrix with an element $B$:

$$[m_\nu, A] = 0, \quad [m_l, B] = 0.$$  \hspace{1cm} (254)

Then, $m_\nu$ and $A$ have the same eigenvectors and therefore the same diagonalizing matrix $V_\nu$:

$$U_\nu^\dagger A U_\nu = A^{\text{diag}},$$
$$U_\nu^\dagger m_\nu U_\nu = m_\nu^{\text{diag}}.$$ \hspace{1cm} (255)

$m_l$ and $B$ also have the same diagonalizing matrix $U_l$. The diagonalizing matrices $U_\nu$ and $U_l$ define the mixing matrix.

Since we want to find elements of the $PSL(2, 7)$ that reproduce the neutrinos mixing matrix, we need to know the three-dimensional representations of the group elements. E.
Floratos and G. Leontaris in [104] derived the three-dimensional representations of the \( PSL(2, p) \) generators. Here, all the elements of the three-dimensional representations of \( PSL(2, 7) \) are derived.

### 8.1 \( PSL(2, 7) \) three-dimensional representations

The unitary, three-dimensional representations of the \( PSL(2, 7) \) group are presented in this section. The elements of the \( PSL(2, p) \) group, where \( p \) is a prime number, can be defined as combinations of powers of two generators \( a \) and \( b \). In a specific representation, they are written as

\[
a = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}; \quad b = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}.
\]

(256)

and they satisfy the relations

\[
a^2 = b^3 = -I = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

(257)

The 3-dimensional representation of \( PSL_2(7) \) is a subgroup of \( SU(3) \). \( PSL_2(3) \) is actually \( A_3 \), while \( PSL_2(5) \) is \( A_5 \). Both \( A_3 \) and \( A_5 \) groups have been studied extensively. The next group, \( PSL_2(11) \), does not have a triplet representations and so we are not interested in.

The number of elements of a \( SL_2(p) \) group, with \( p \) prime, is \( p(p^2 - 1) \), while that of the corresponding projective \( PSL_2(p) \) is \( p(p^2 - 1)/2 \). So, the \( PSL_2(7) \) has 168 elements and it is a simple discrete subgroup of \( SU(3) \). The three-dimensional representations of \( PSL_2(7) \) were constructed in [104] and they satisfy the conditions

\[
a^2 = b^3 = (ab)^7 = [a, b] = 1
\]

(258)

where \([a, b] = a^{-1}b^{-1}ab\).

All the elements of the three-dimensional unitary representations of \( PSL_2(7) \) were produced from the two generators using the GAP system for computational discrete algebra available on the web [108]. The 3x3 matrices of these two generators have both the form of the latin squares [109], with their elements carrying some phase.

\[
A_{[3]} = \begin{bmatrix} \rho_1 & -\rho_2 & -\rho_3 \\ -\rho_2 & \rho_3 & \rho_1 \\ -\rho_3 & \rho_1 & \rho_2 \end{bmatrix}, \quad B_{[3]} = \begin{bmatrix} \rho_1\eta^{-1} & \rho_2\eta^2 & \rho_3\eta^{-2} \\ \rho_2\eta^2 & \rho_3\eta^{-2} & \rho_1\eta^2 \\ \rho_3\eta^2 & \rho_1\eta^{-2} & \rho_2\eta^3 \end{bmatrix}.
\]

(259)
<table>
<thead>
<tr>
<th>Order</th>
<th>character</th>
<th>#</th>
<th>Tag</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>−1</td>
<td>21</td>
<td>$e_{l_2}$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>56</td>
<td>$e_{l_3}$</td>
</tr>
<tr>
<td>4</td>
<td>+1</td>
<td>42</td>
<td>$e_{l_4}$</td>
</tr>
<tr>
<td>7</td>
<td>$-\frac{1}{2} \pm i\frac{\sqrt{7}}{2}$</td>
<td>48</td>
<td>$e_{l_7}$</td>
</tr>
</tbody>
</table>

Table 2: The order, character, and the number of elements of the conjugacy classes of $PSL(2,7)$

where

$$\rho_1 = -\frac{2}{\sqrt{7}} \cos \frac{\pi}{14}, \quad \rho_2 = -\frac{2}{\sqrt{7}} \cos \frac{3\pi}{14}, \quad \rho_3 = -1 - \rho_1 - \rho_2, \quad \eta = e^{2\pi i/7}.$$  \hspace{1cm} (260)

$\rho_1, \rho_2, \rho_3$ satisfy the cubic equation

$$x^3 + x^2 - \frac{1}{7} = 0,$$  \hspace{1cm} (261)

and the relation

$$\rho_2 = 7\rho_1^3 - 3\rho_1.$$  \hspace{1cm} (262)

The conjugacy classes of the group, along with their properties, are shown in Table 8.1.

### 8.2 Properties of the representation matrices

This section is about properties of the latin square matrices and their relation to the $PSL(2,7)$ elements. A $3 \times 3$ latin square matrix can have one of the following forms:

$$M_1 = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_2 & r_3 & r_1 \\ r_3 & r_1 & r_2 \end{bmatrix}, \quad M_2 = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_3 & r_1 & r_2 \\ r_2 & r_3 & r_1 \end{bmatrix},$$  \hspace{1cm} (263)

and their permutations. From these two, only the first type appears in $PSL(2,7)$. Requiring orthogonality ($M_1^2 = 1$) we get

$$r_1^2 + r_2^2 + r_3^2 = 1$$

$$r_1r_2 + r_1r_3 + r_2r_3 = 0,$$  \hspace{1cm} (264)

while $\det M_1 = 1$ leads to

$$r_1 + r_2 + r_3 = -1.$$  \hspace{1cm} (265)
Since the character of the matrix is $-1$, it is an element of the conjugacy class $e l_2$, as one can see in Table 8.1. $r_1, r_2, r_3$ satisfy the algebraic equation

$$x^3 + x^2 - q = 0,$$  \(266\)

where $q = r_1 r_2 r_3$. For the roots to be real, it has to be $0 < q < \frac{4}{27}$. This is true for $PSL(2, 7)$ since $q = \frac{1}{7} < \frac{4}{27}$.

Generalizing the matrix in order its elements to be complex, we introduce phase terms:

$$M = \begin{bmatrix} r_1 e^{ic_1} & r_2 e^{ic_2} & r_3 e^{ic_3} \\ r_2 e^{ic_4} & r_3 e^{ic_5} & r_1 e^{ic_6} \\ r_3 e^{ic_7} & r_1 e^{ic_8} & r_2 e^{ic_9} \end{bmatrix}. \quad (267)$$

For $M$ to be unitary and its determinant value to be 1, the conditions (264,265) are still needed. The number of independent phases also is reduced from 9 to 4:

$$M = \begin{bmatrix} r_1 e^{ic_1} & r_2 e^{i(c_1-c_2+c_5)} & r_3 e^{i(c_1-c_2+c_3)} \\ r_2 e^{i(c_1-c_2+c_5)} & r_3 e^{i(c_1-c_2+c_3)} & r_1 e^{i(c_2-c_1-c_3)} \\ r_3 e^{-i(c_3+c_5)} & r_1 e^{i(c_2-c_1-c_3)} & r_2 e^{-i(c_1+c_5)} \end{bmatrix}. \quad (268)$$

A lot of the $PSL(2, 7)$ elements now produced by GAP, can be written in this form substituting $r_1, r_2, r_3$ with $\rho_1, \rho_2, \rho_3$ (Eq. (260)) and adding phases closely related to the set of the seventh roots of unity. The characteristic equation for $M$ is

$$x^3 - \text{tr}(M) x^2 + \text{tr} M^* x - 1 = 0. \quad (269)$$

The roots of the characteristic equation are the eigenvalues of the representation matrices. The character of the order seven elements is $\text{Tr} M = -\frac{1}{2} - i \frac{\sqrt{7}}{2}$. The eigenvalues found solving the characteristic equation (269) are

$$\exp \left[ \frac{10\pi i}{7} \right], \exp \left[ \frac{6\pi i}{7} \right], \exp \left[ \frac{12\pi i}{7} \right], \quad (270)$$

in accordance to the group elements table.

For the order two elements it turns out that $c_1 = c_5 = 0$. So $M$ becomes

$$M = \begin{bmatrix} r_1 & r_2 e^{ic_2} & r_3 e^{ic_3} \\ r_2 e^{-ic_2} & r_3 & r_1 e^{i(c_3-c_2)} \\ r_3 e^{-ic_3} & r_1 e^{i(c_2-c_3)} & r_2 \end{bmatrix}. \quad (271)$$

Next, for the general matrix $M$, information on the allowed values for $c_1$ and $c_5$ can be extracted by taking the system of equations

$$r_1 e^{ic_1} + r_2 e^{-i(c_1+c_5)} + r_3 e^{ic_5} = \text{Tr} M \quad (272)$$

$$r_1 + r_2 + r_3 = -1 \quad (273)$$

$$r_1 r_2 + r_2 r_3 + r_3 r_1 = 0 \quad (274)$$
and substituting the character \( \text{Tr} M \) of the corresponding conjugacy class. Parametrizing the phases as

\[
c_1 = \frac{2\pi}{7} n, \quad c_5 = \frac{2\pi}{7} m
\]

and taking \( q = r_1 r_2 r_3 = \frac{1}{7} \) we get only integer values for \( n, m \). These values are symmetric under the interchange of \( m \) and \( n \). Notice that the value, \( q = \frac{1}{7} \) identifies \( r_1, r_2, r_3 \) with \( \rho_1, \rho_2, \rho_3 \) respectively.

### 8.3 The PSL(2, 7) elements that produce a neutrino mixing matrix compatible with the experimental data

The observed neutrino oscillations indicate non-zero neutrino masses \( m_\nu \) and non-zero \( \theta_{ij} \) mixing angles in the lepton sector, as explained in the previous chapter. The general form of the lepton mixing matrix can be written as

\[
U = U_l^T U_\nu = \begin{pmatrix}
c_{12} c_{13} & c_{13} s_{12} & s_{13} e^{-i\delta} \\
-c_{23} s_{12} - c_{12} s_{13} s_{23} e^{i\delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} & -s_{13} e^{i\delta} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{23} s_{12} s_{13} - c_{12} s_{23} s_{13} e^{i\delta} & c_{13} c_{23}
\end{pmatrix}
\]

(276)

where \( c_{ij} \equiv \cos \theta_{ij} \) and \( s_{ij} \equiv \sin \theta_{ij} \). The 3\( \sigma \) range of the three mixing angles we used in [3], is

\[
\sin^2 \theta_{12} = [0.25 - 0.35], \quad \sin^2 \theta_{23} = [0.38 - 0.62], \quad \sin^2 \theta_{13} = [0.0185 - 0.0246].
\]

(277)

We are going to examine whether our results on the masses and mixing matrices can match the experimental data. First, we suppose that the neutrino and charged lepton mass matrices commute with the elements of the \( \text{PSL}(2, 7) \). Then we write the diagonalizing matrices \( U \) for these elements. The eigenvalues now that are the building blocks of the resulting diagonal matrices can be arranged in \( 3! \) different ways in the matrix and the same holds for the eigenvectors that are the columns of the diagonalizing matrix. In order to conduct a complete search, we build all the mixing matrices that are generated by combining all the diagonalizing matrices. This is done numerically. Then we keep only the mixing matrices that are compatible with the experimental and observational data:

\[
0.136 < |U_{13}| < 0.157, \quad 0.499 < |U_{12}| < 0.595, \quad 0.615 < |U_{23}| < 0.785
\]

(278)

The search showed that only some elements of order two for the neutrino mass matrix give a mixing matrix that obeys these limits. As for the charged leptons mass matrix, a number of order 3 and order 7 elements give acceptable mixing matrices. These results
are shown in the tables 8.3 and 8.3. The inverse elements are also accepted but they are not presented in the tables, since the inverse of an order 2 element is itself and the inverse of an order 3 or 7 element is easily calculated.

Now we have a comment on the order 2 elements. The eigenvalues of these matrices are $(1, -1, -1)$ and that means that there is a degenerate 2-dimensional subspace implying that the eigenvectors related to the degenerate eigenvalue are not uniquely defined. If $v_1, v_2, v_3$ are eigenvectors corresponding to the $(1, -1, -1)$ eigenvalues an equally good choice would be $v_1, \tilde{v}_2, \tilde{v}_3$, where

$$
\begin{bmatrix}
\tilde{v}_2 \\
\tilde{v}_3
\end{bmatrix} = \begin{bmatrix} e^{i\varphi_1} \cos \varphi & -e^{i\varphi_2} \sin \varphi \\
  e^{i\varphi_1} \sin \varphi & e^{i\varphi_2} \cos \varphi \end{bmatrix} \begin{bmatrix} v_2 \\
 v_3
\end{bmatrix}
$$

(279)

for arbitrary $\varphi, \varphi_1, \varphi_2$. This matrix defines a $U(2)$ rotation for $v_2$ and $v_3$ that leaves the representation matrices invariant.
8.4 Specific examples of $PSL(2, 7)$ elements generating a mixing matrix that obeys the experimental bounds

In the following subsections we present two cases in which the produced mixing matrix is compatible with the observational and experimental data.

8.4.1 The $e_l^2\,(16)$, $e_l^3\,(5)$ pair

The $e_l^2\,(16)$ and $e_l^3\,(5)$ matrices are

$$
e_l^2\,(16) = \begin{bmatrix} r_3 & -r_1 & -r_2 \\ -r_1 & r_2 & r_3 \\ -r_2 & r_3 & r_1 \end{bmatrix},$$

$$
e_l^3\,(5) = \begin{bmatrix} 0 & 0 & -e^{\frac{6\pi i}{7}} \\ e^{-\frac{2\pi i}{7}} & 0 & 0 \\ 0 & e^{-\frac{4\pi i}{7}} & 0 \end{bmatrix} \quad (280)$$

The normalized eigenvectors of $e_l^2\,(16)$ are

$$v_2[1] = \left[ \frac{1}{2}s - \frac{\sqrt{3}}{2} \sqrt{\frac{3}{3} - s^2} \right], \quad v_2[2] = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2[3] = \begin{bmatrix} \sqrt{\frac{2}{3} - s^2} + \frac{1}{2} \sqrt{\frac{3}{3} - s^2} \\ \frac{\sqrt{3}}{2} s + \frac{1}{2} \sqrt{\frac{2}{3} - s^2} \\ -\frac{\sqrt{3}}{2} s + \frac{1}{2} \sqrt{\frac{2}{3} - s^2} \end{bmatrix} \quad (281)$$

and they correspond to the eigenvalues $+1, -1, -1$ respectively, while $s$ is

$$s = \sqrt{2} \frac{(r_1 - r_2)}{\sqrt{(1 - 3r_3)^2 + 3 (r_2 - r_1)^2}} \approx -0.815 \quad (282)$$

The normalized eigenvectors of $e_l^3\,(5)$ are

$$v_3[1] = \frac{1}{\sqrt{3}} \begin{bmatrix} -e^{\frac{6\pi i}{7}} \\ e^{\frac{4\pi i}{7}} \\ 1 \end{bmatrix}, \quad v_3[2] = \frac{1}{\sqrt{3}} \begin{bmatrix} -e^{\frac{4\pi i}{7}} \\ -e^{\frac{6\pi i}{7}} \\ 1 \end{bmatrix}, \quad v_3[3] = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{\frac{11\pi i}{7}} \\ -e^{\frac{5\pi i}{7}} \\ 1 \end{bmatrix} \quad (283)$$

and they correspond to the eigenvalues $1, e^{\frac{2\pi i}{7}}, e^{\frac{4\pi i}{7}}$.

We remind here that a diagonalizing matrix is built by the eigenvectors arranged in columns and there are $3!$ such possible arrangements. For the mixing matrices generated by the diagonalizing matrices of $e_l^2\,(16)$ and $e_l^3\,(5)$, it turns out that only two combinations conform to the experimental and observational data:

$$U_1 = [v_3[3], \ v_3[2], \ v_3[1]]^\dagger \cdot [\tilde{v}_2[2], \ v_2[1], \ \tilde{v}_2[3]]$$

$$U_2 = [v_3[3], \ v_3[1], \ v_3[2]]^\dagger \cdot [\tilde{v}_2[2], \ v_2[1], \ \tilde{v}_2[3]] \quad (284)$$
where $\tilde{v}_2[2]$, $\tilde{v}_2[3]$ are as in Eq. (279). For the value of $s$ given in Eq. (282) it is

$$
\left\| v_3[2]^\dagger \cdot v_2[1] \right\| = \left\| v_3[1]^\dagger \cdot v_2[1] \right\| = \left\| v_3[3]^\dagger \cdot v_2[1] \right\| = \frac{1}{\sqrt{3}} \approx 0.5773
$$

(285)

and for this reason, the moduli of the second column elements for both mixing matrices $U_1$ and $U_2$ have a value of $1/\sqrt{3}$. For $\varphi = 0$ $U_1$ and $U_2$ become

$$
U_1 = \begin{bmatrix}
0.80217 e^{0.5667 i} & 0.57735 e^{2.3948 i} & 0.152283 e^{-1.27039 i} \\
0.36647 e^{0.106487 i} & 0.57735 e^{-0.8735 i} & 0.729634 e^{-0.3499 i} \\
0.471405 e^{-1.6582 i} & 0.57735 e^{3.05416 i} & 0.666667 e^{0.635302 i}
\end{bmatrix}
$$

(286)

$$
U_2 = \begin{bmatrix}
0.80217 e^{0.5667 i} & 0.57735 e^{2.3948 i} & 0.152283 e^{-1.27039 i} \\
0.471405 e^{-1.6582 i} & 0.57735 e^{3.05416 i} & 0.666667 e^{0.635302 i} \\
0.36647 e^{0.106487 i} & 0.57735 e^{-0.8735 i} & 0.729634 e^{-0.3499 i}
\end{bmatrix}
$$

(287)

Figure (12) shows the range of $\varphi$ values that provide accordance with the observational and experimental data.

**8.4.2 The $el_2(16)$, $el_7(23)$ pair**

In this subsection the $PSL(2,7)$ element that commutes with the charged lepton mass matrix is of the seventh-order. The matrices of $el_2(10)$, $el_7(23)$ are

$$
el_2(10) = \begin{bmatrix}
r_1 & -r_2 & -r_3 \\
-r_2 & r_3 & r_1 \\
-r_3 & r_1 & r_2
\end{bmatrix},
$$

$$
el_7(23) = \begin{bmatrix}
r_1 e^{\frac{4\pi i}{7}} & r_2 e^{\frac{\pi i}{7}} & r_3 e^{-\frac{5\pi i}{7}} \\
r_2 e^{-\frac{5\pi i}{7}} & r_3 e^{-\frac{4\pi i}{7}} & r_1 e^{\frac{2\pi i}{7}} \\
r_3 e^{\frac{2\pi i}{7}} & r_1 e^{-\frac{2\pi i}{7}} & r_2 e^{\frac{3\pi i}{7}}
\end{bmatrix}.
$$

(288)
The eigenvectors of $e_l (10)$ are

$$v_2 [1] = \begin{bmatrix} \sqrt{2/3} s + \frac{1}{2} \sqrt{2/3 - s^2} \\ -\sqrt{2/3} s + \frac{1}{2} \sqrt{2/3 - s^2} \end{bmatrix}, \quad v_2 [2] = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad v_2 [3] = \begin{bmatrix} \frac{1}{2} s - \sqrt{3} \frac{2}{3} \sqrt{2/3 - s^2} \\ \frac{1}{2} s + \sqrt{3} \frac{2}{3} \sqrt{2/3 - s^2} \end{bmatrix}$$

(289)

and they correspond to the eigenvalues $1, -1, -1$ respectively. Here $s$ is

$$s = \sqrt{\frac{2}{3} \frac{(2 + 3r_3)}{(3 + r_1 - r_2)^2 + (2 + 3r_3)^2}} \approx 0.732.$$ 

(290)

The normalized eigenvectors of $e_l (23)$ are

$$v_7 [1] = \begin{bmatrix} r_3 e^{\frac{4\pi i}{7}} \\ r_1 e^{\frac{2\pi i}{7}} \\ r_2 e^{\frac{6\pi i}{7}} \end{bmatrix}, \quad v_7 [2] = \begin{bmatrix} r_2 e^{\frac{6\pi i}{7}} \\ r_3 e^{\frac{2\pi i}{7}} \\ -r_1 \end{bmatrix}, \quad v_7 [3] = \begin{bmatrix} -r_1 \\ r_3 e^{-\frac{6\pi i}{7}} \\ r_2 e^{-\frac{2\pi i}{7}} \end{bmatrix}$$

(291)

and they correspond to the eigenvalues $e^{\frac{6\pi i}{7}}, e^{\frac{10\pi i}{7}}, e^{\frac{12\pi i}{7}}$. It turns out that the diagonalizing matrices of all order seven elements can be written as latin square matrices. Yet, we do not know why this happens and also they are not elements of the $PSL(2,7)$ group. The mixing matrices compatible with data are

$$U_1 = [v_7 [1]^t, v_7 [2]^t, v_7 [3]^t] \cdot [v_2 [1], \bar{v}_2 [3], \bar{v}_2 [2]]$$

(292)

$$U_2 = [v_7 [1]^t, v_7 [3]^t, v_7 [2]^t] \cdot [v_2 [1], \bar{v}_2 [3], \bar{v}_2 [2]].$$

(293)

For $\varphi = 0$ the mixing matrices are completely out. For $\phi = \frac{2\pi}{7}$ they are

$$U_1 = \begin{bmatrix} 0.814857 e^{-\frac{3\pi i}{7}} & 0.558406 e^{\frac{\pi i}{7}} & 0.15532 e^{\frac{3\pi i}{7}} \\ 0.362646 e^{\frac{4\pi i}{7}} & 0.700416 e^{\frac{\pi i}{7}} & 0.61471 e^{-\frac{4\pi i}{7}} \\ 0.452212 e^{\frac{5\pi i}{7}} & 0.444523 e^{-\frac{3\pi i}{7}} & 0.773242 e^{-\frac{5\pi i}{7}} \end{bmatrix}$$

$$U_2 = \begin{bmatrix} 0.814857 e^{-\frac{3\pi i}{7}} & 0.558406 e^{\frac{\pi i}{7}} & 0.15532 e^{\frac{3\pi i}{7}} \\ 0.452212 e^{-\frac{5\pi i}{7}} & 0.444523 e^{-\frac{3\pi i}{7}} & 0.773242 e^{-\frac{5\pi i}{7}} \\ 0.362646 e^{\frac{4\pi i}{7}} & 0.700416 e^{\frac{\pi i}{7}} & 0.61471 e^{\frac{4\pi i}{7}} \end{bmatrix}. $$

(294)

The range of $\delta \varphi \equiv \varphi - \frac{2\pi}{7}$ that falls within the experimental bounds is very narrow:

$$-0.0028 < \delta \varphi < +0.03$$

(295)
Figure 13: The experimentally compatible range of $\sin \theta_{13}$ as function of $\varphi$, for the $el_2 (10)$, $el_7 (23)$ pair. The orange line is the upper experimental bound while the blue is the lower one.

8.5 Conclusions

We conducted a complete investigation about whether any mixing matrices produced with the assumption that the neutrino and the charged leptons mass matrices commute with the $PSL(2,7)$ elements obey the experimental and observational data. We found such mixing matrices for a number of order 2 elements commuting with neutrino mass matrix and of order 3 and 7 elements commuting with charged leptons mass matrix. Two types of matrices are present in the results for the $(el_3, el_2)$ combination and also two for the $(el_7, el_2)$ combination.

$(el_3, el_2)$ allowed pairs reproduce a number of $A_4$ subgroups of $PSL(2,7)$. It is known that the $3 \times 3$ representation of $A_4$ elements has middle column elements with moduli equal to $\frac{1}{\sqrt{3}}$, which is also a characteristic of a generalized tri-bi-maximal mixing matrix [110]. So, this $\frac{1}{\sqrt{3}}$ value is also present in the $(el_3, el_2)$ mixing matrix middle column. The allowed values of the free parameter $\varphi$ are concentrated around the value $\varphi = 0$, which corresponds to the eigenvector $\frac{1}{\sqrt{3}}(-1, 1, 1)$. A more general form is $\frac{1}{\sqrt{3}}(e^{i\varphi_3}, e^{i(\varphi_3-\varphi_2)}, 1)$ and it is independent of $\rho_1, \rho_2, \rho_3$ that characterize $PSL(2,7)$.

For the $(el_7, el_2)$ combinations, the allowed values of $\varphi$ are around $\frac{2\pi}{7}$. This is the phase that corresponds to the seventh root of 1: $e^{\frac{2\pi i}{7}}$. We also mention that these pairs do not form any subgroup of the $PSL(2,7)$ and for this reason, the values of the middle column elements are not identical.
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